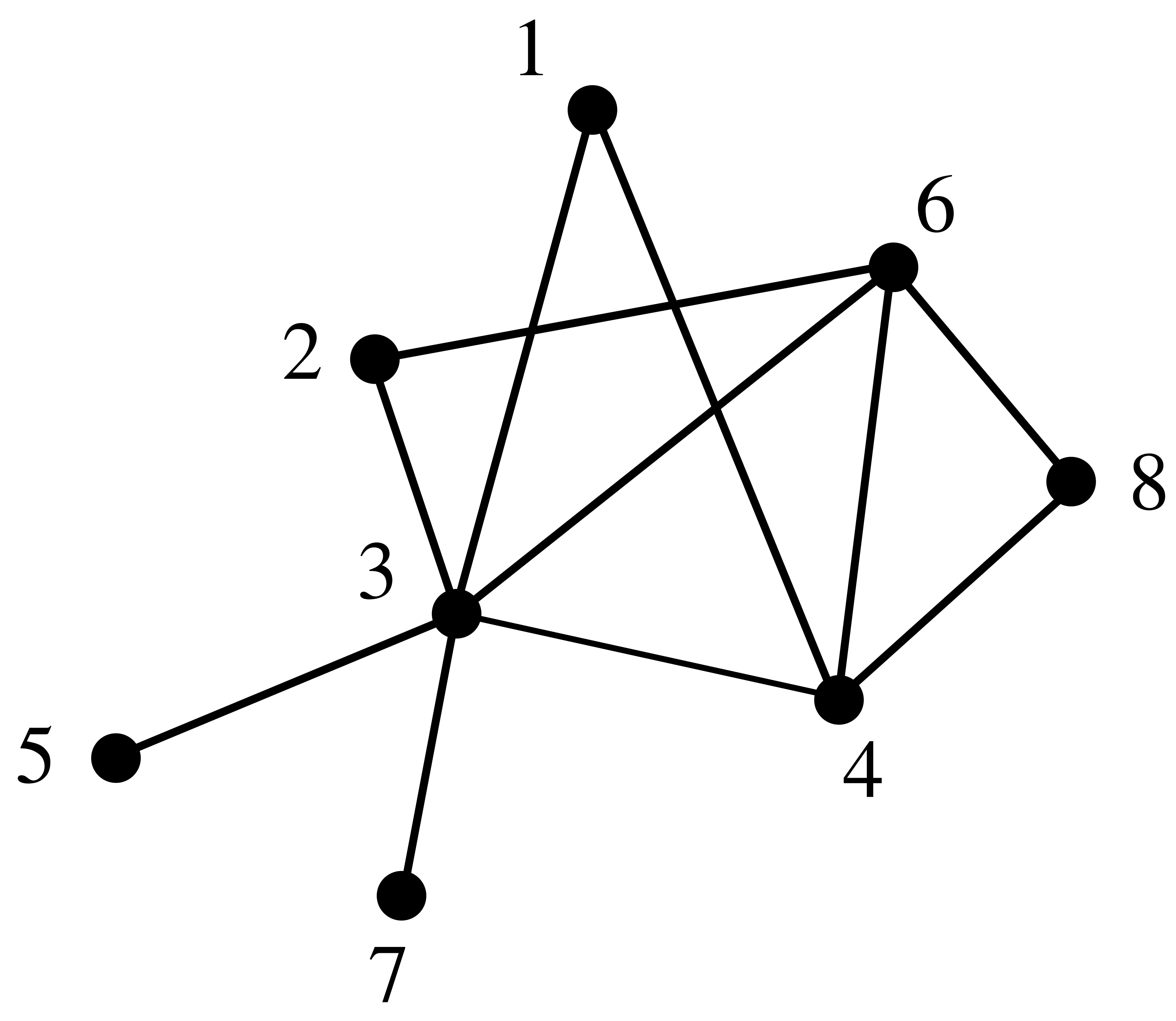
Lecture 20

Bipartite Maximum Matching

Source: Introduction to Algorithms, CLRS and Kleinberg & Tardos

Matching

Matching



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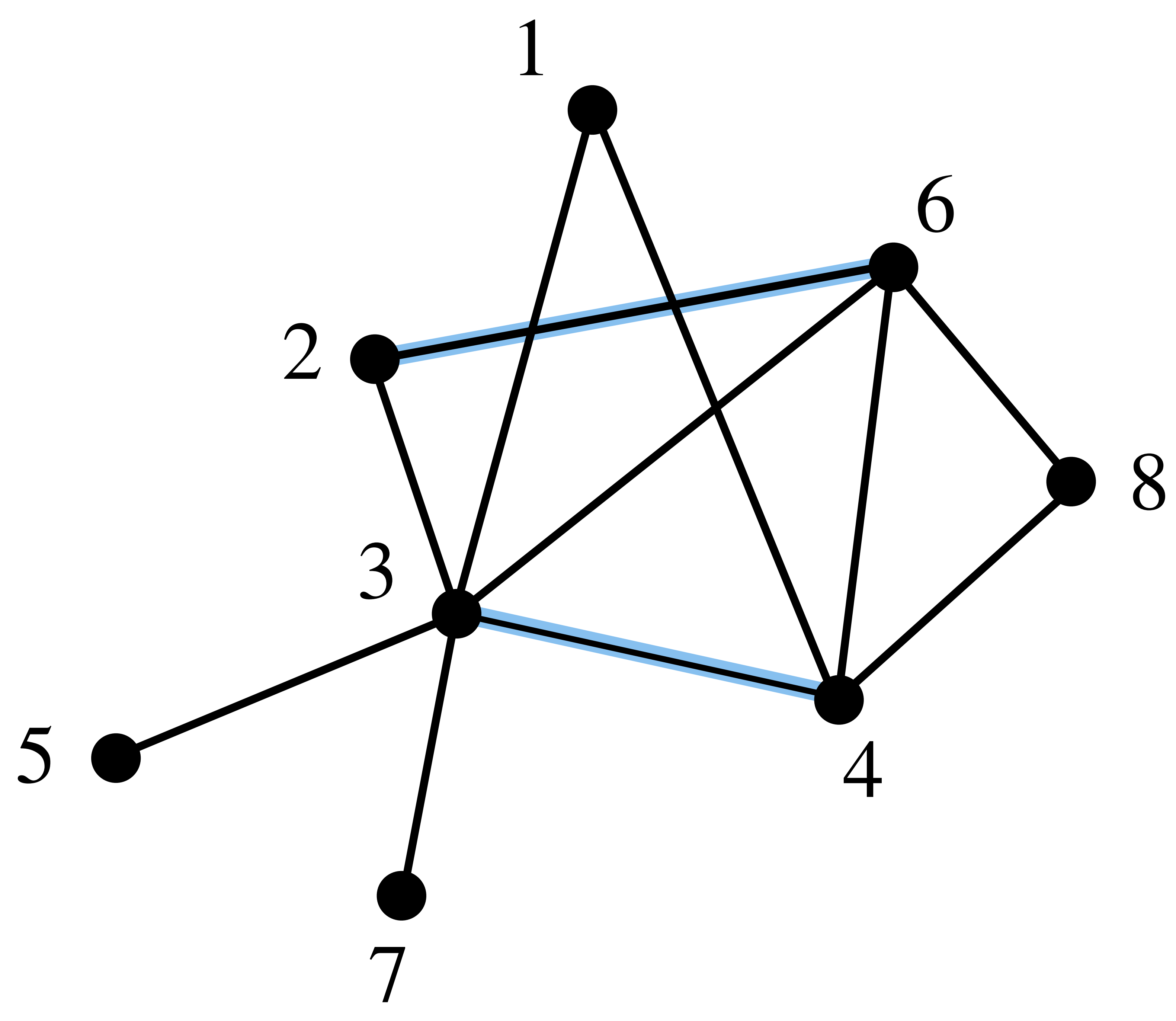

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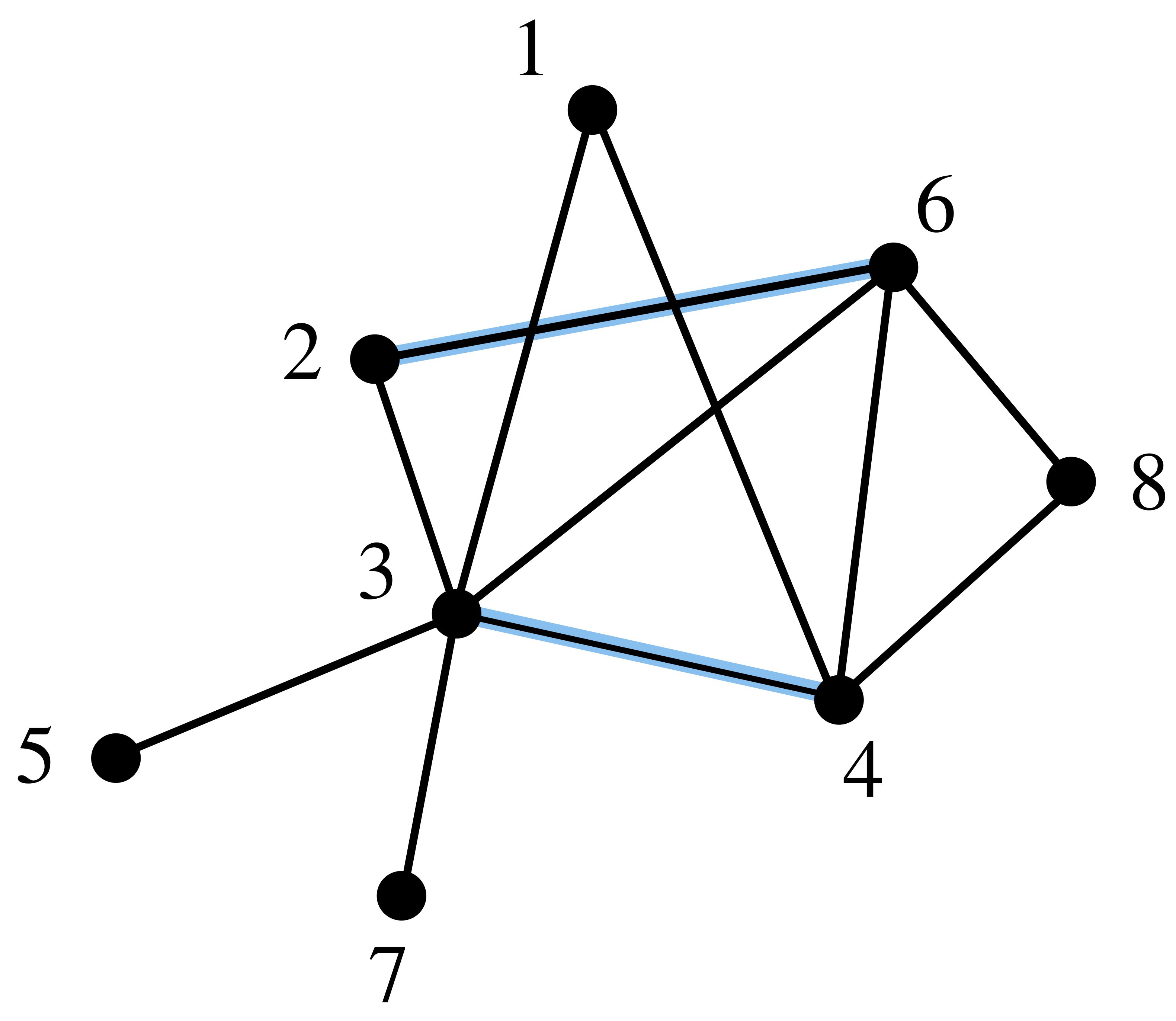

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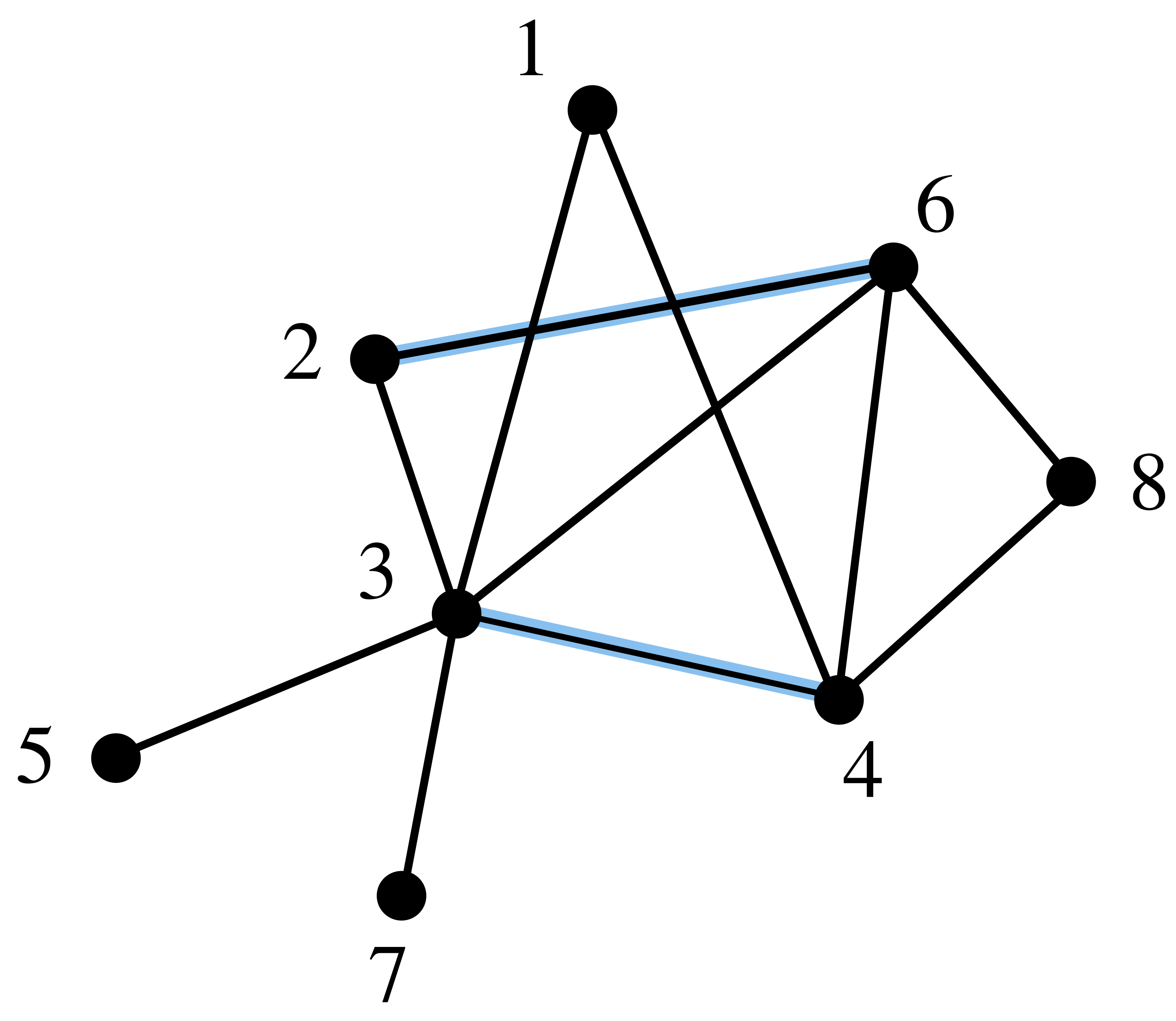

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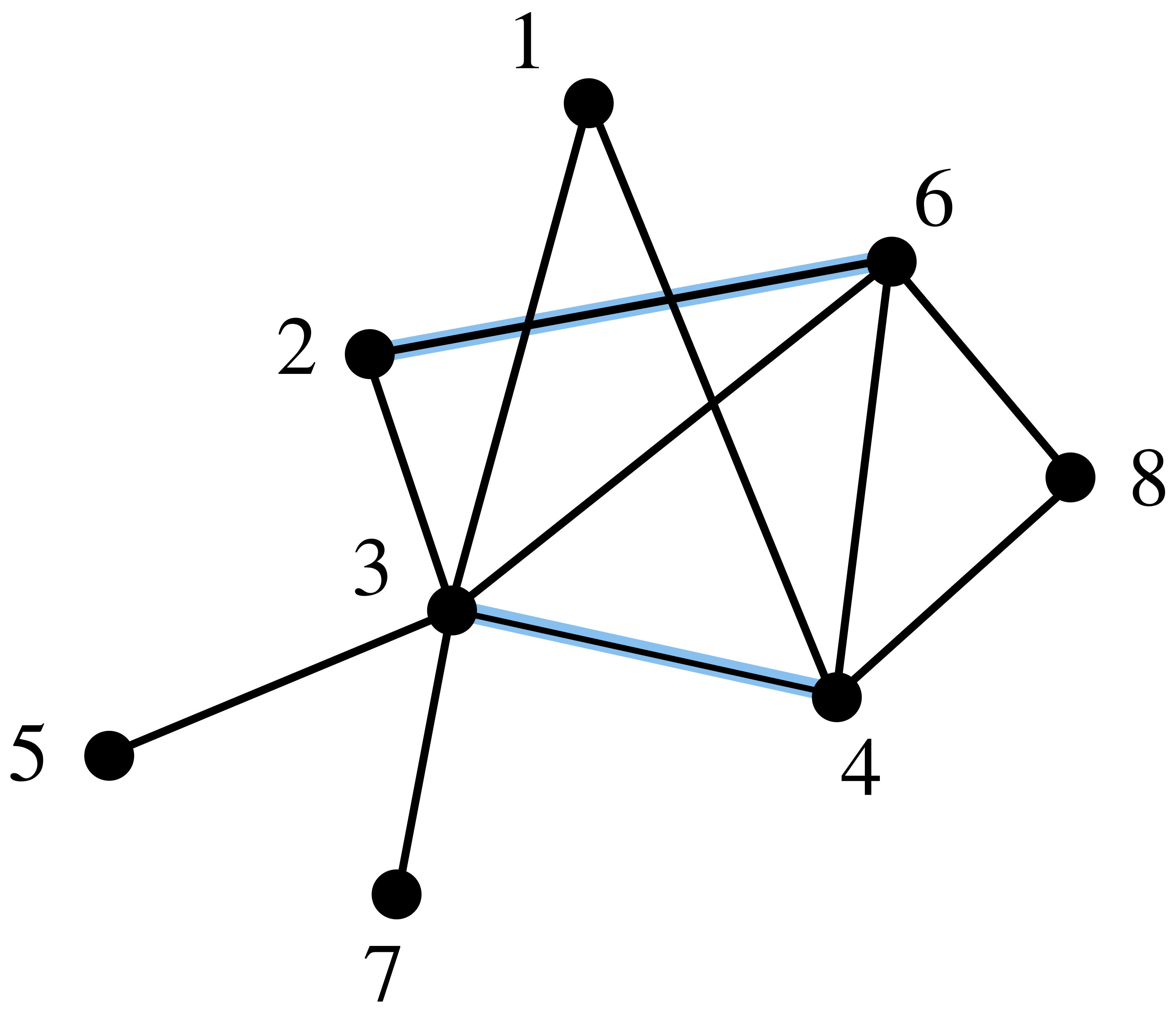

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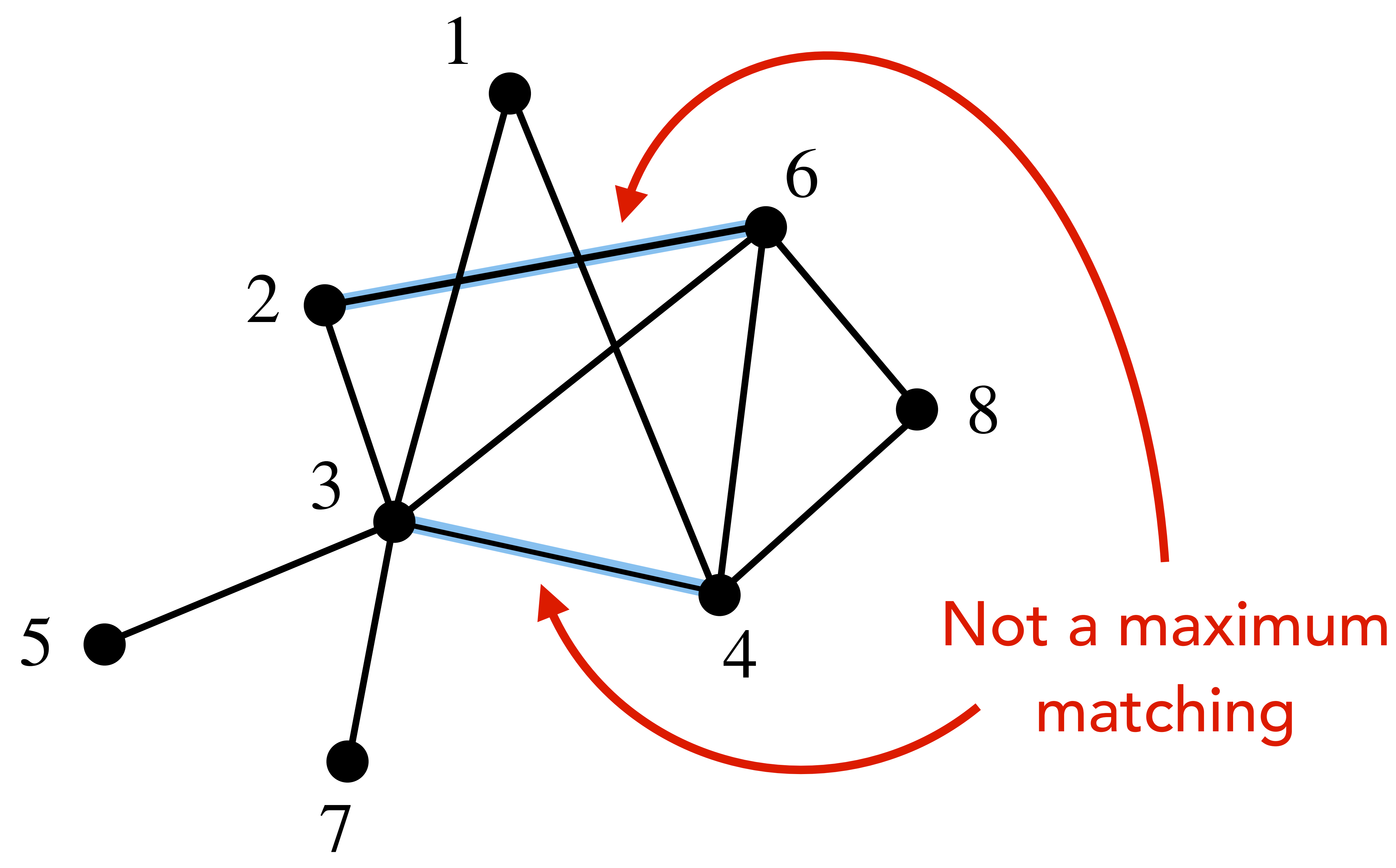

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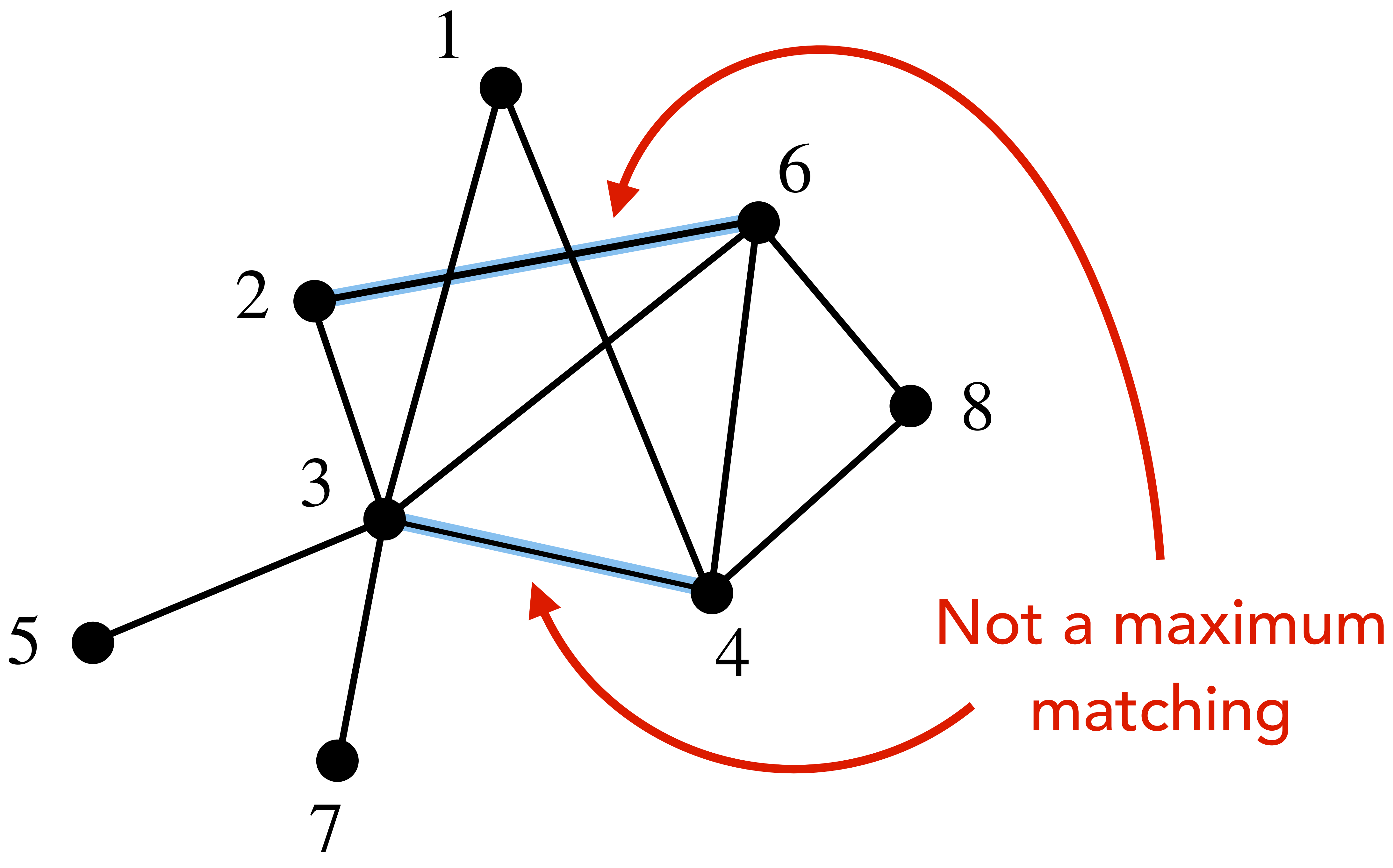

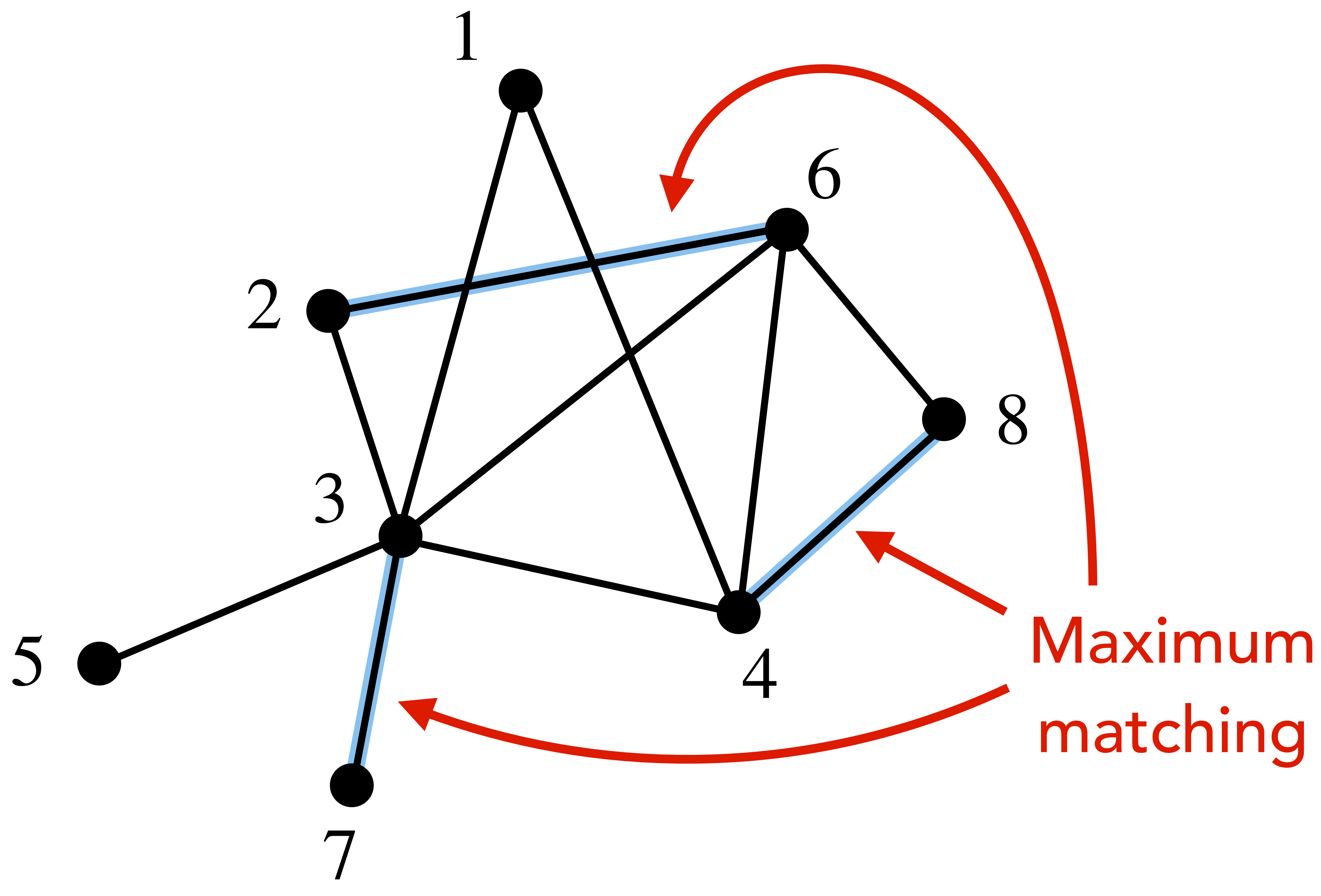

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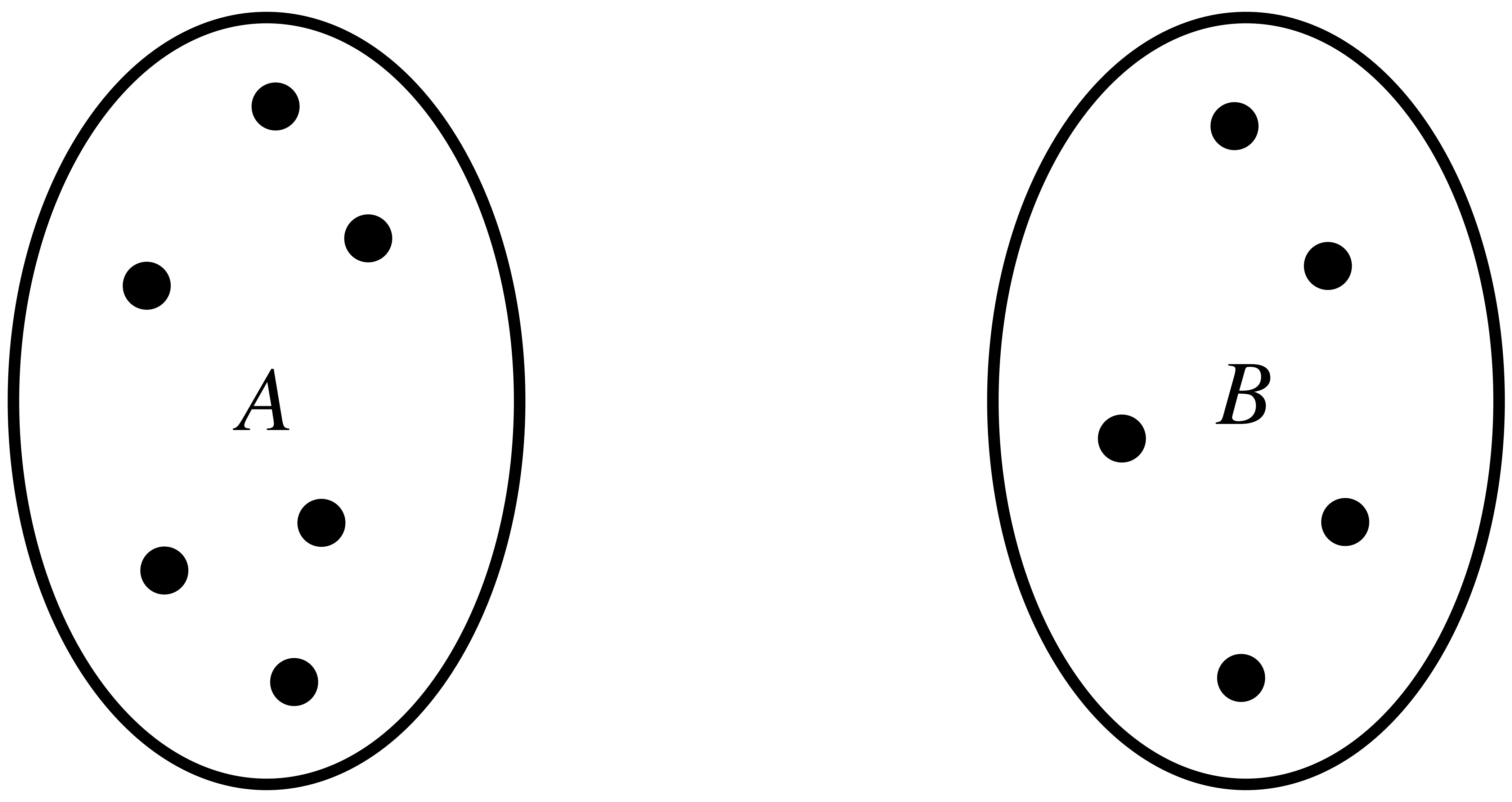

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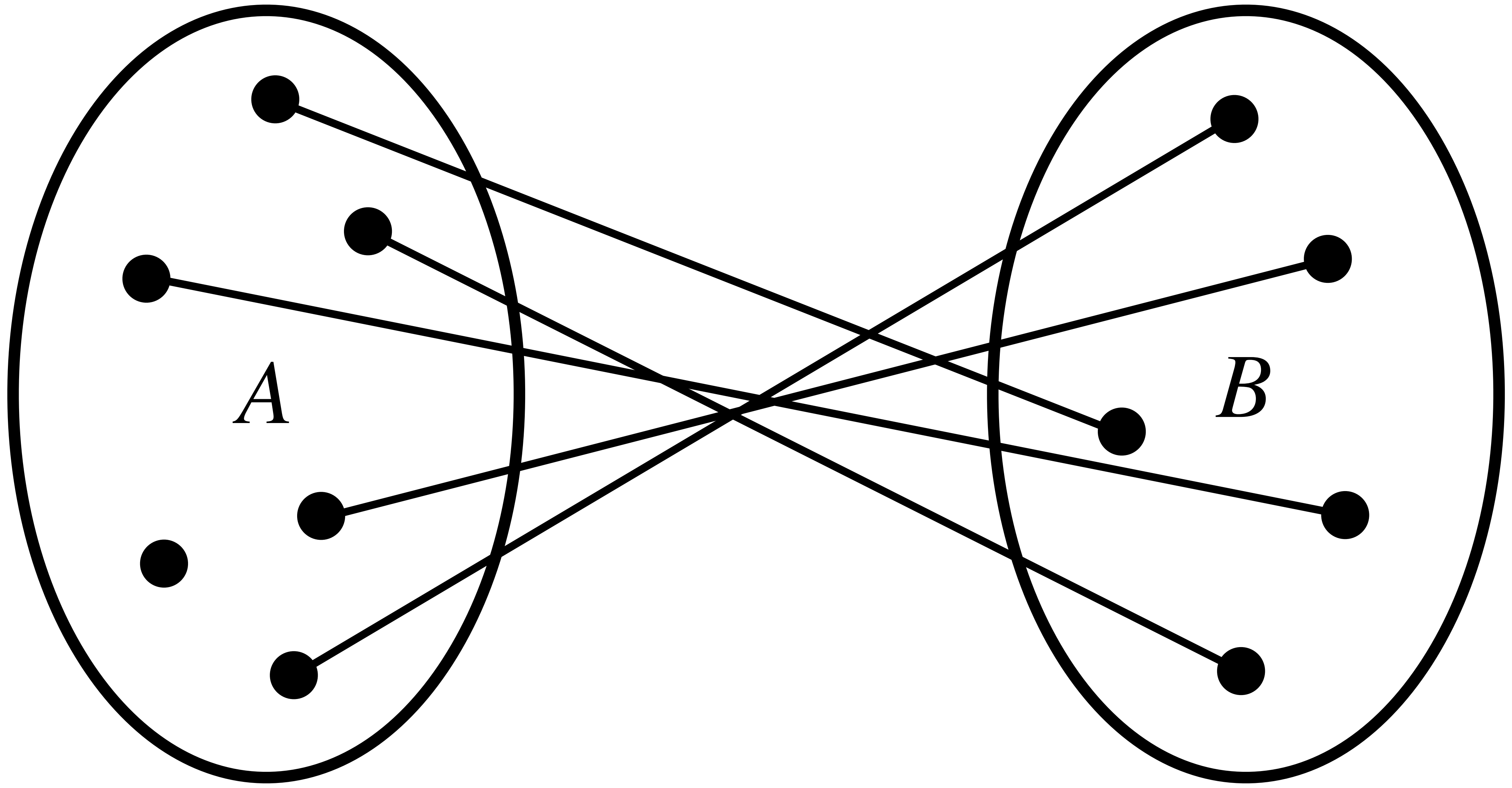

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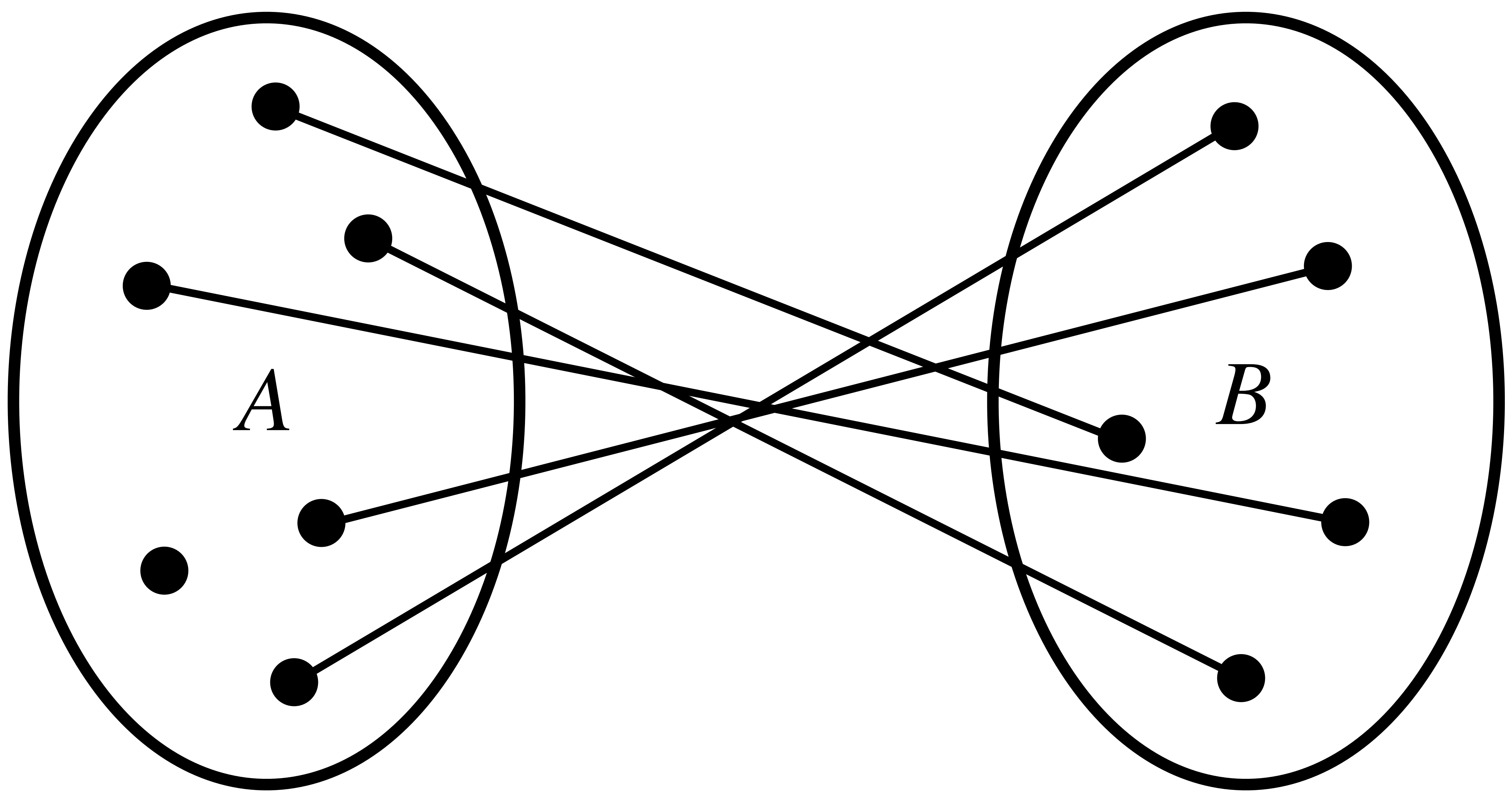

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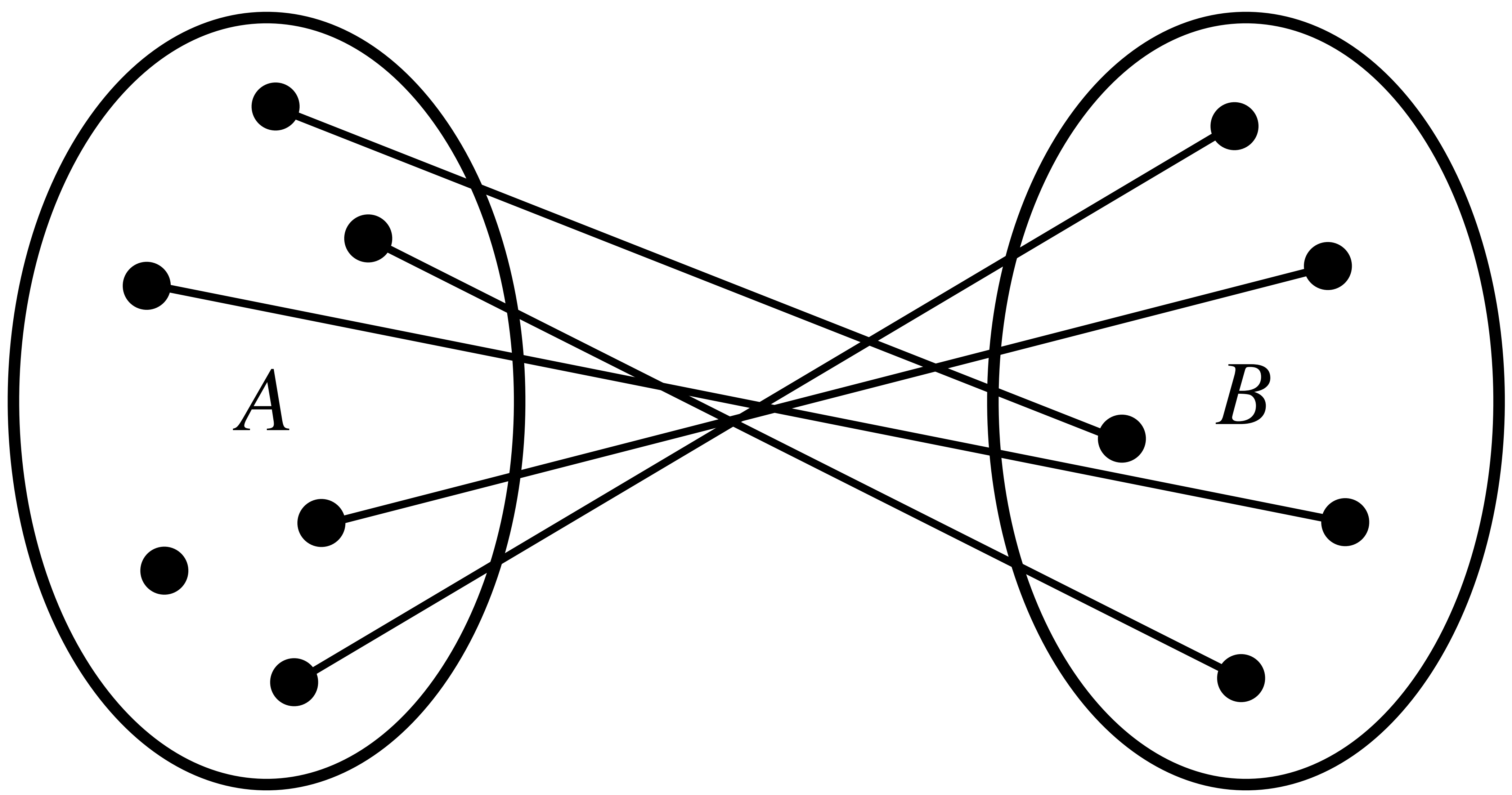

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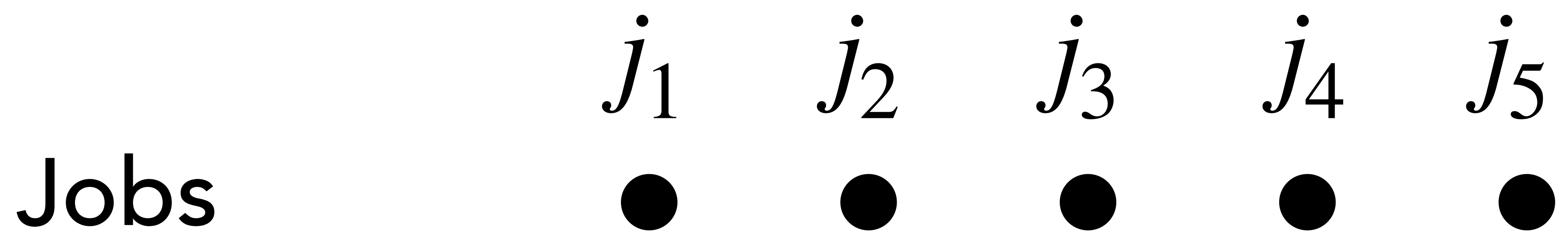

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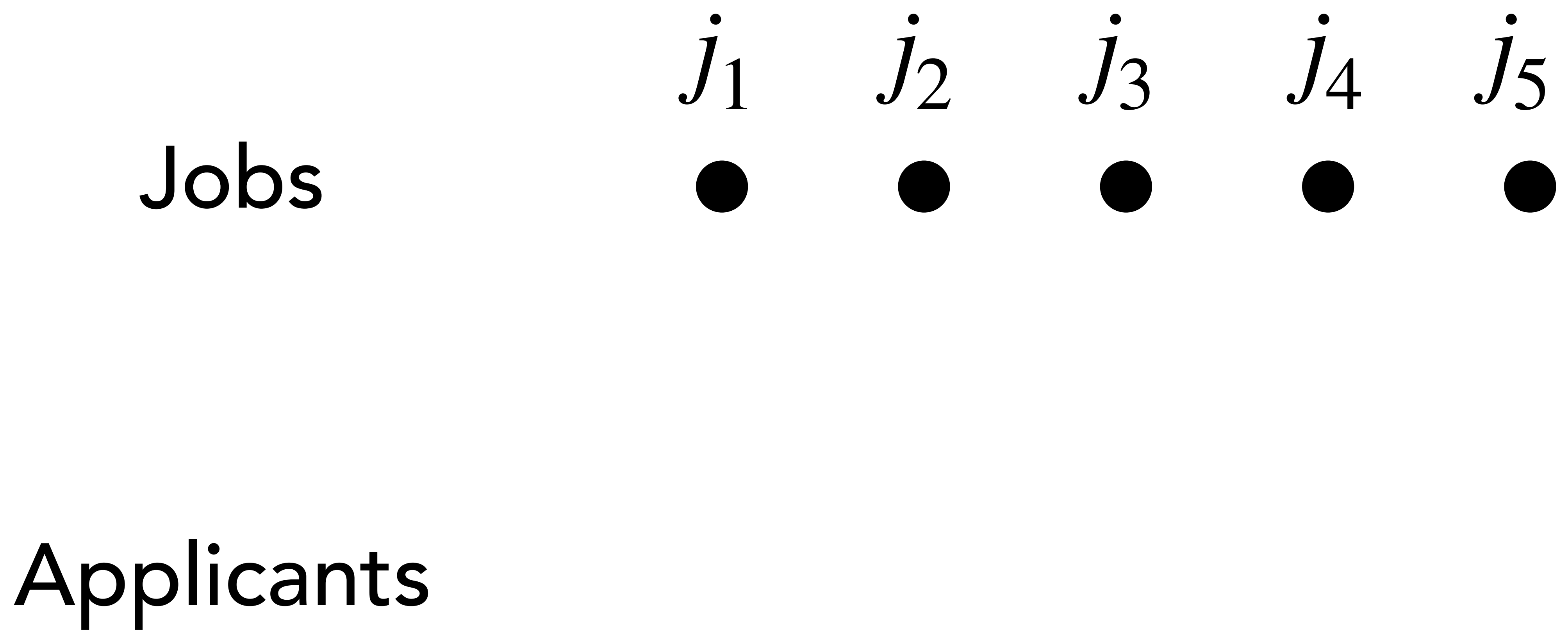

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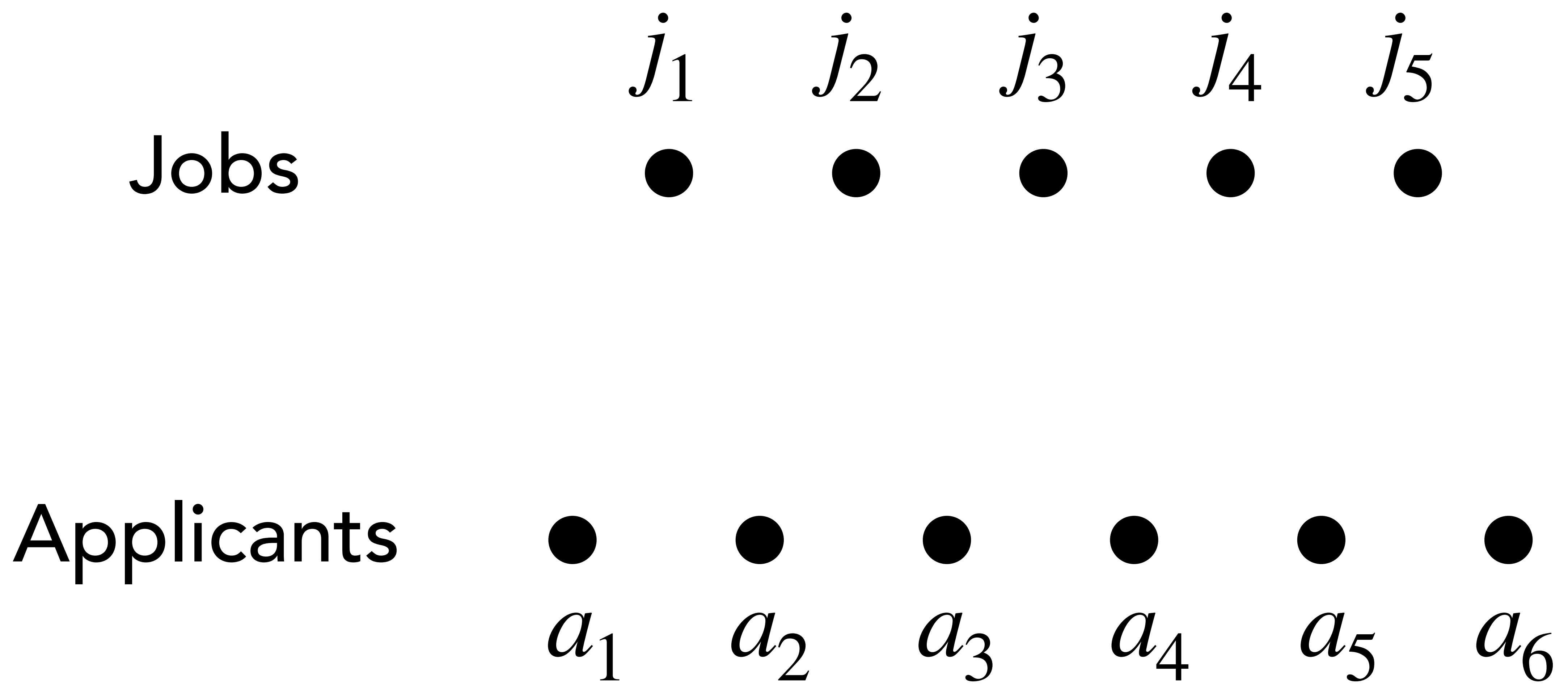

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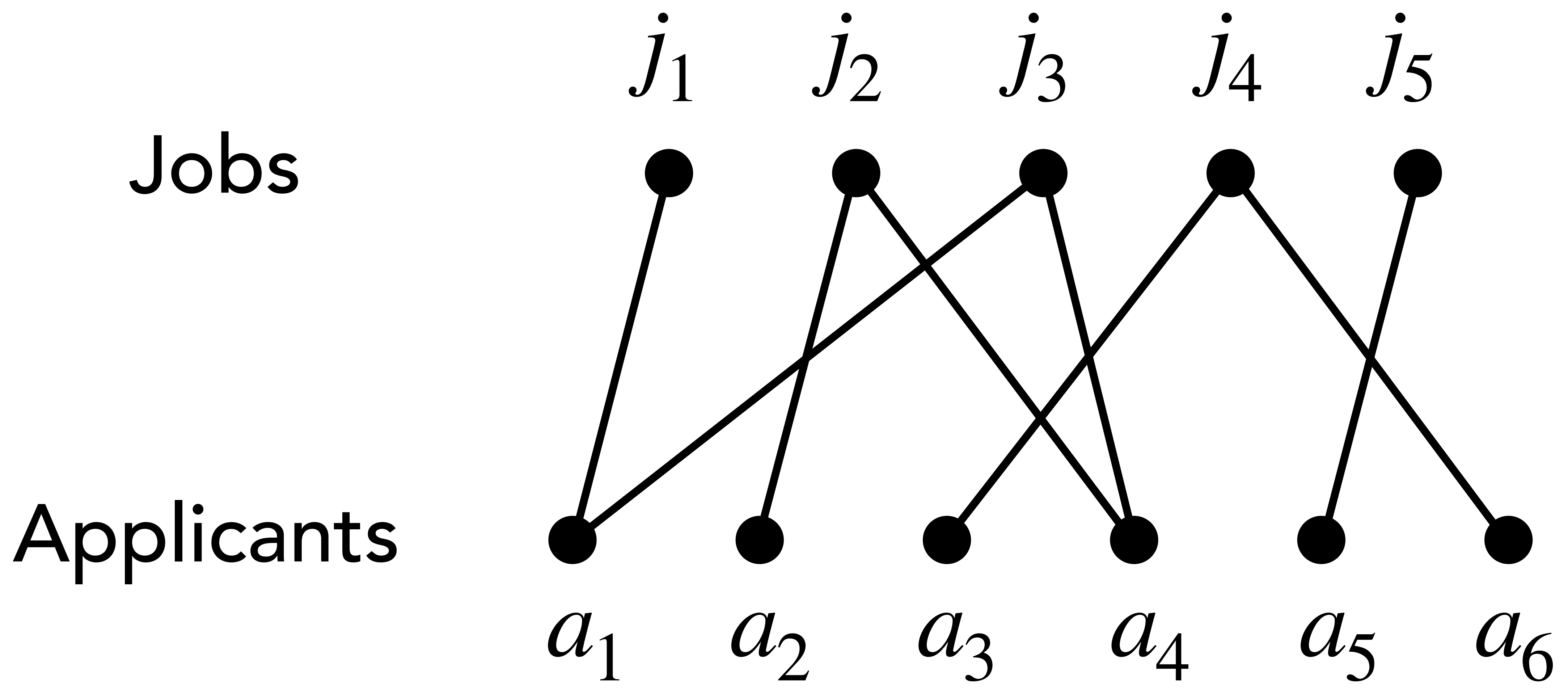

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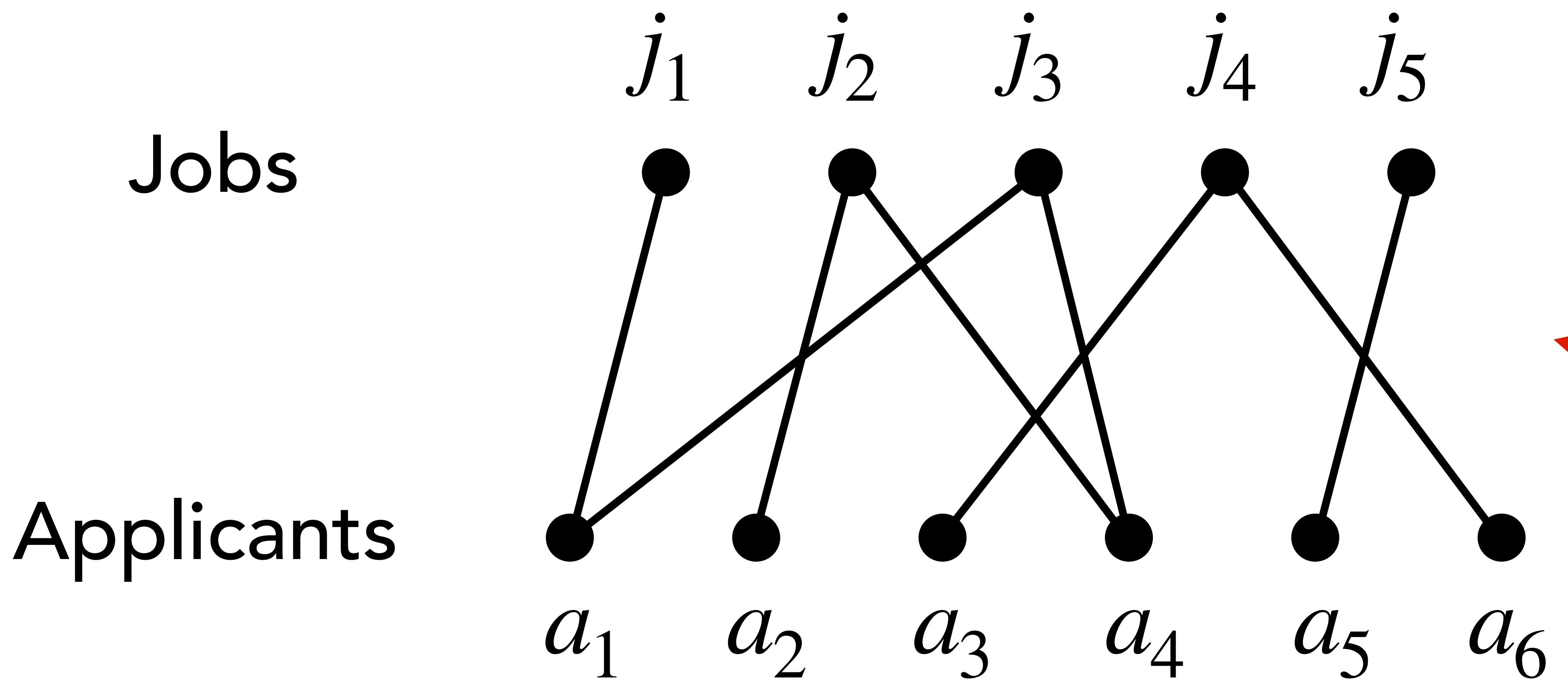

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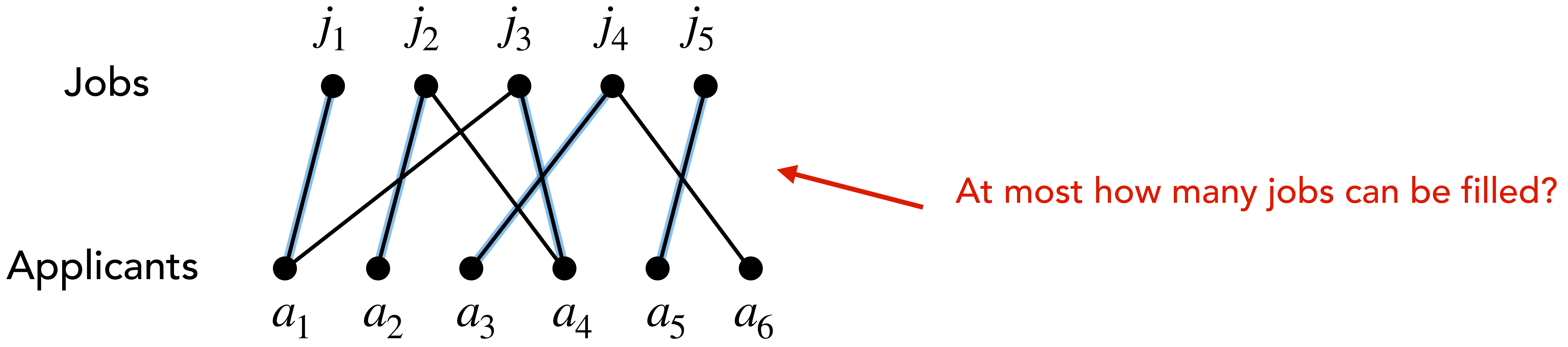

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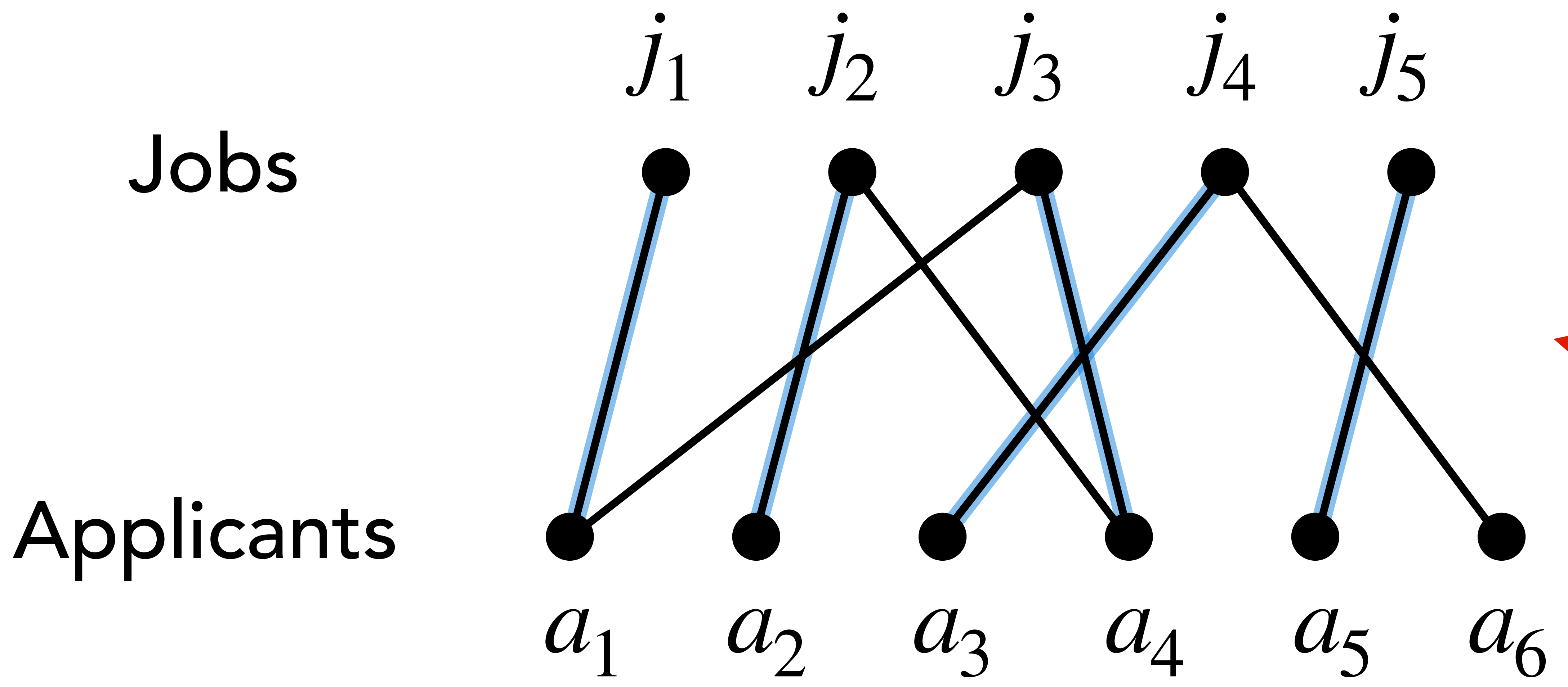

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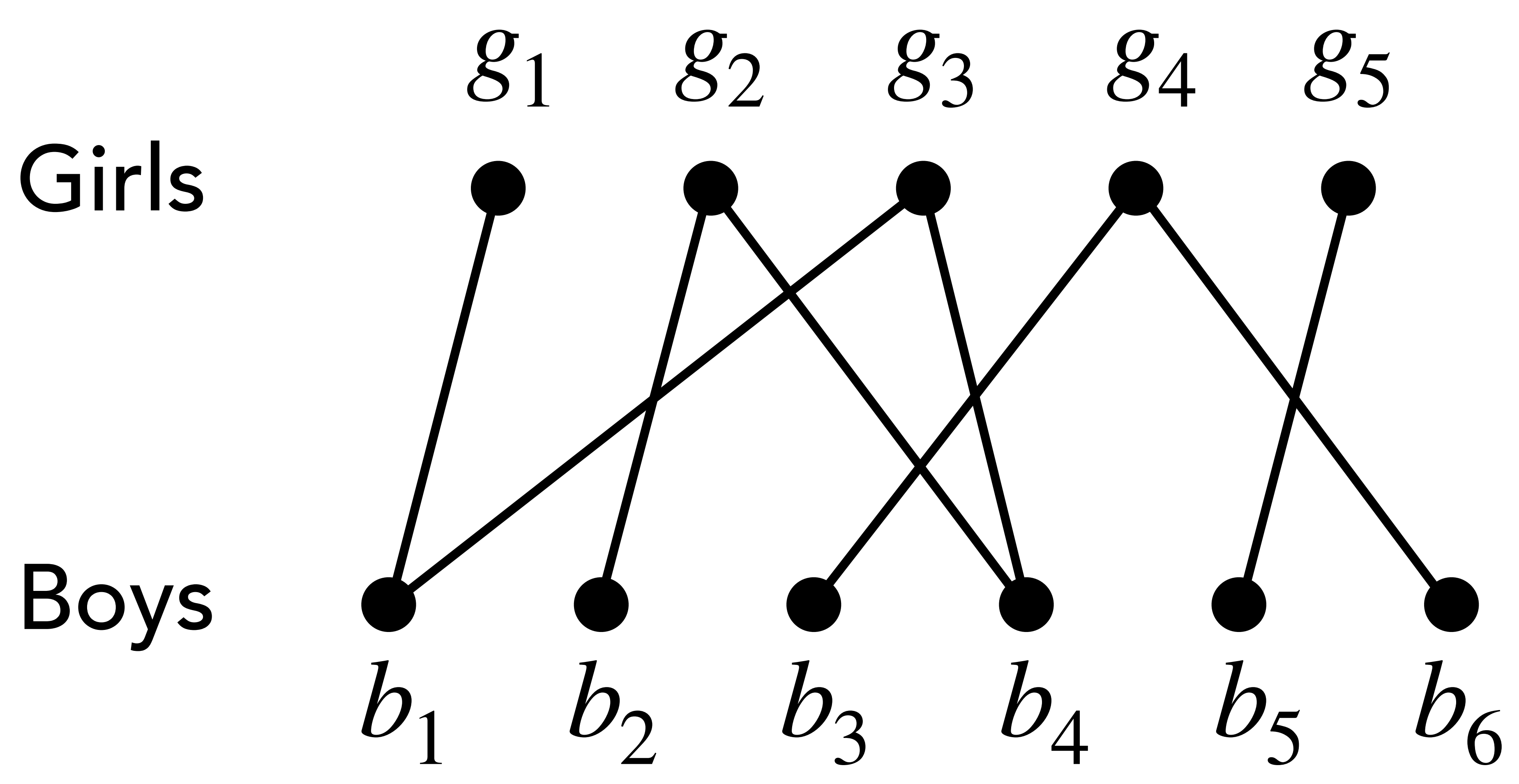

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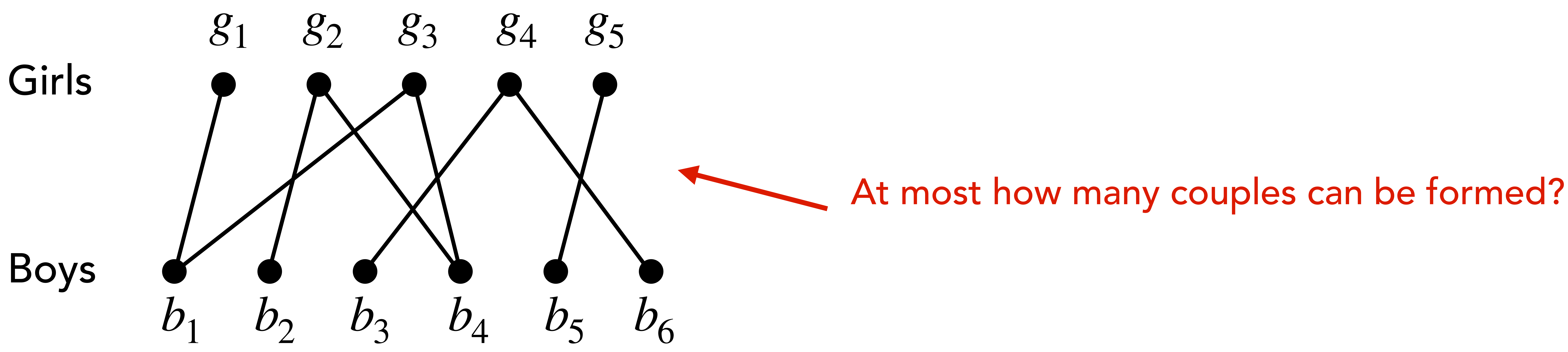

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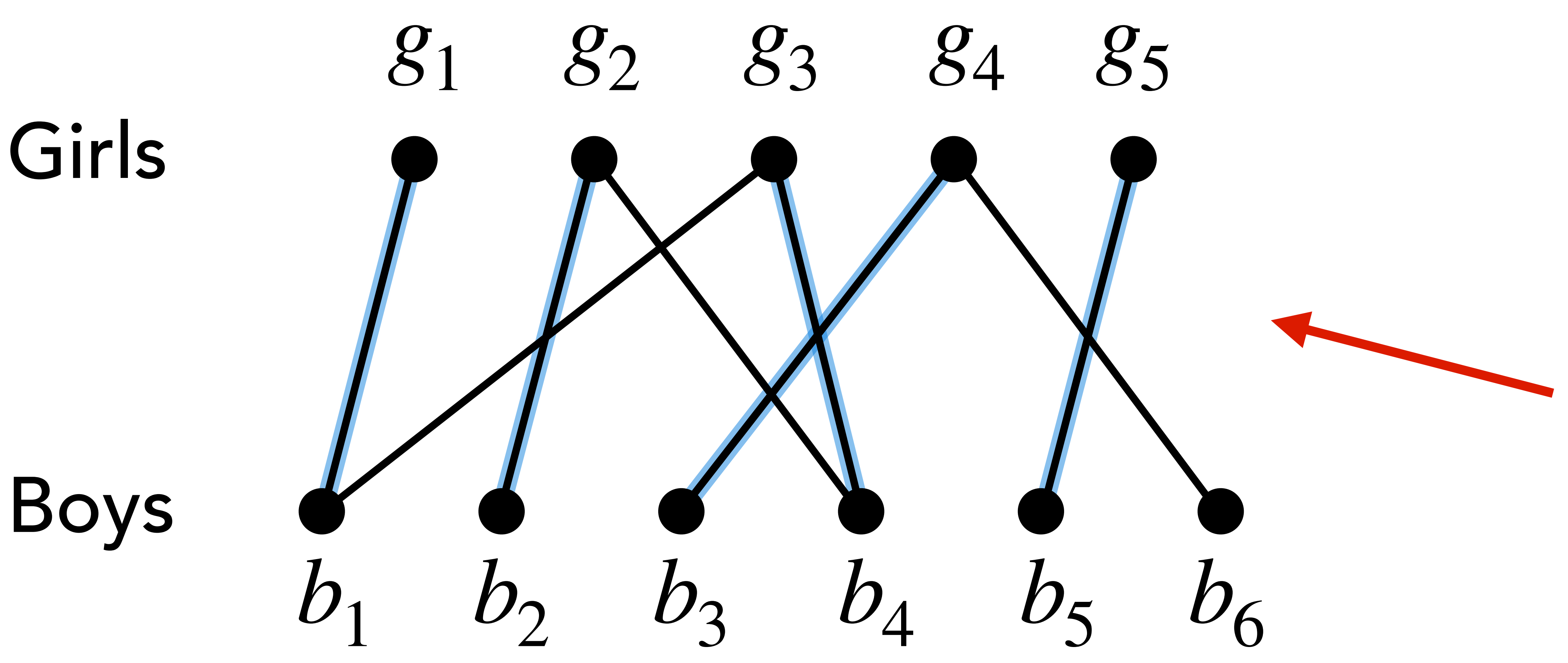

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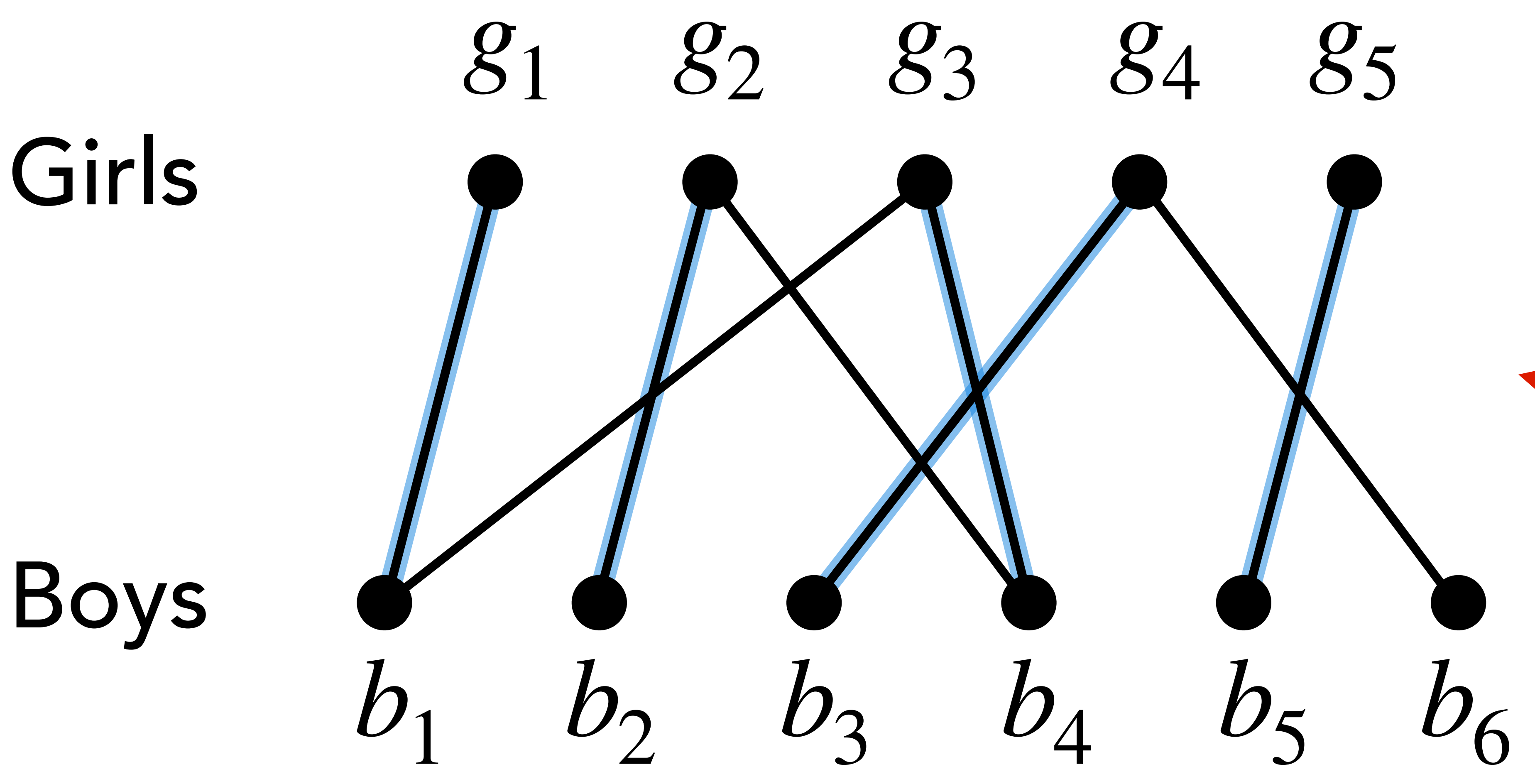

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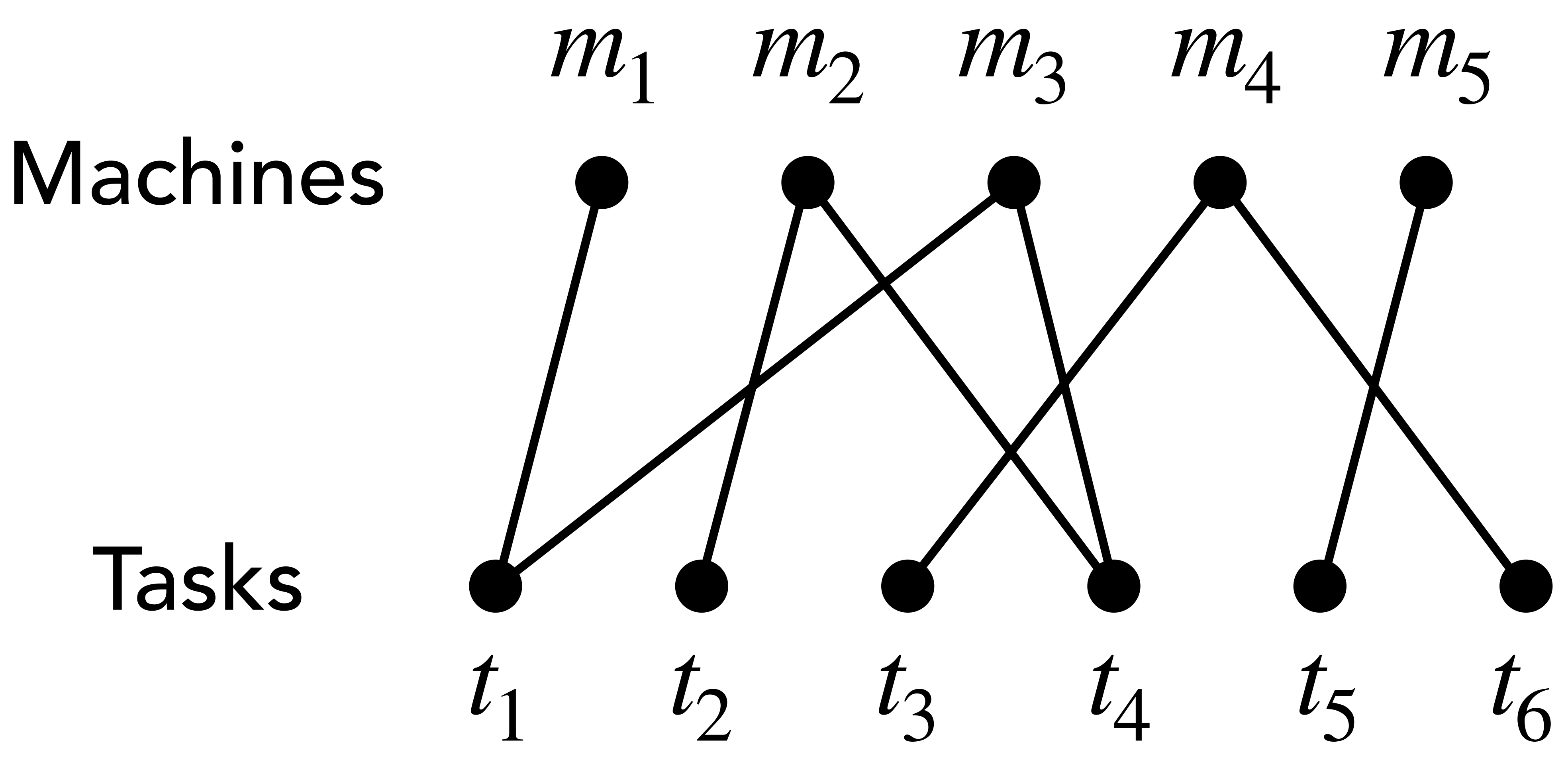

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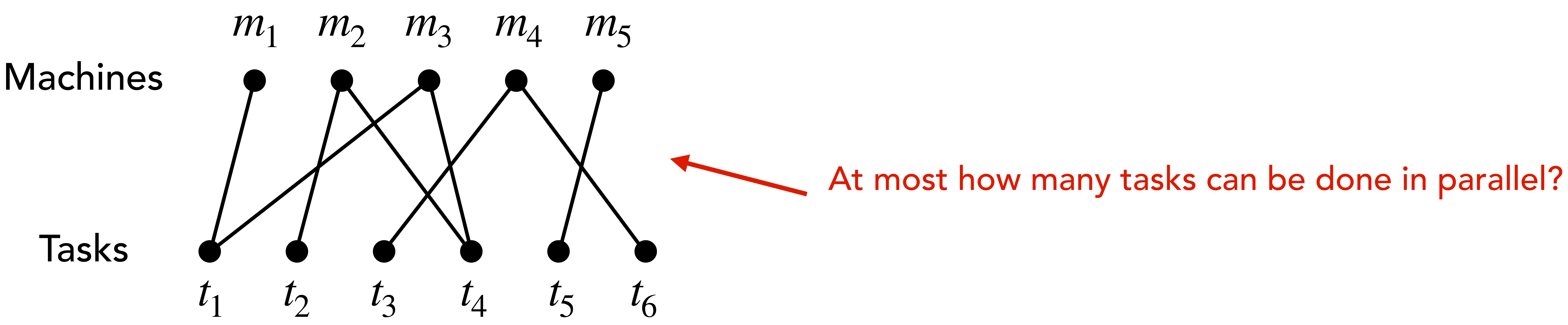

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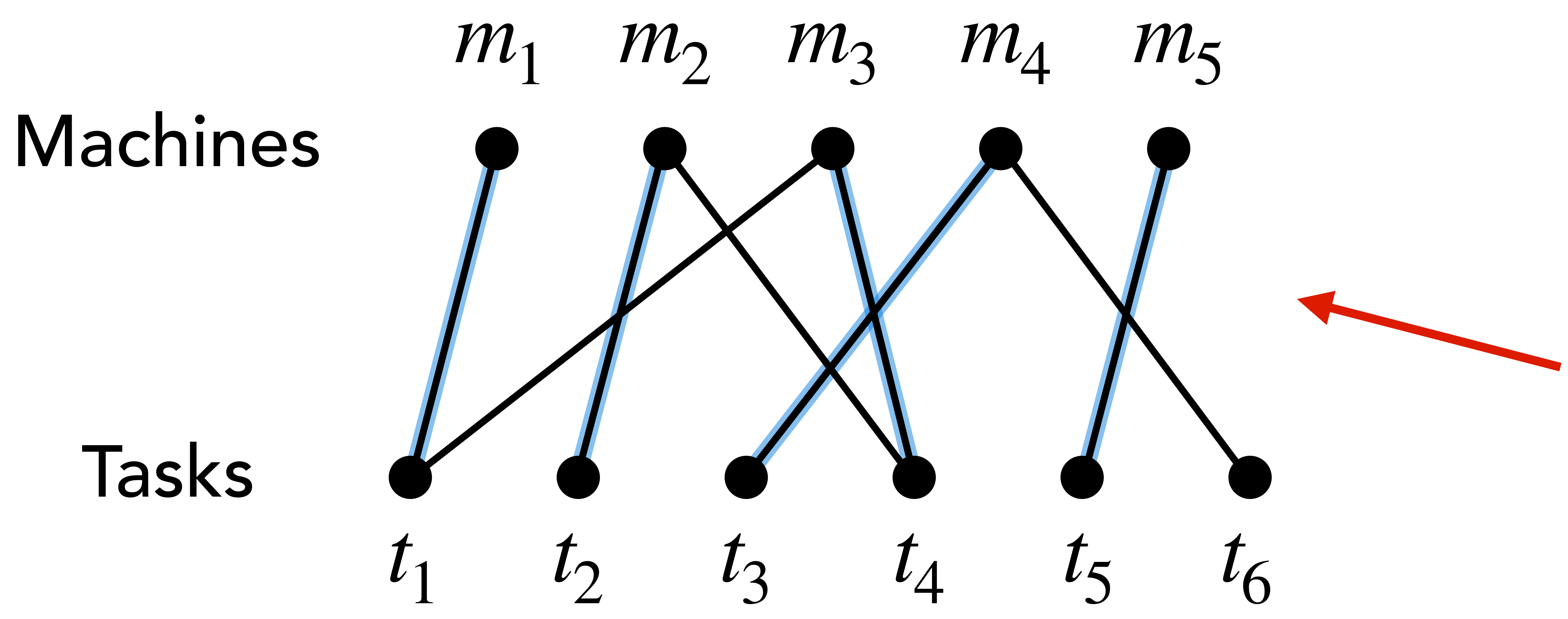

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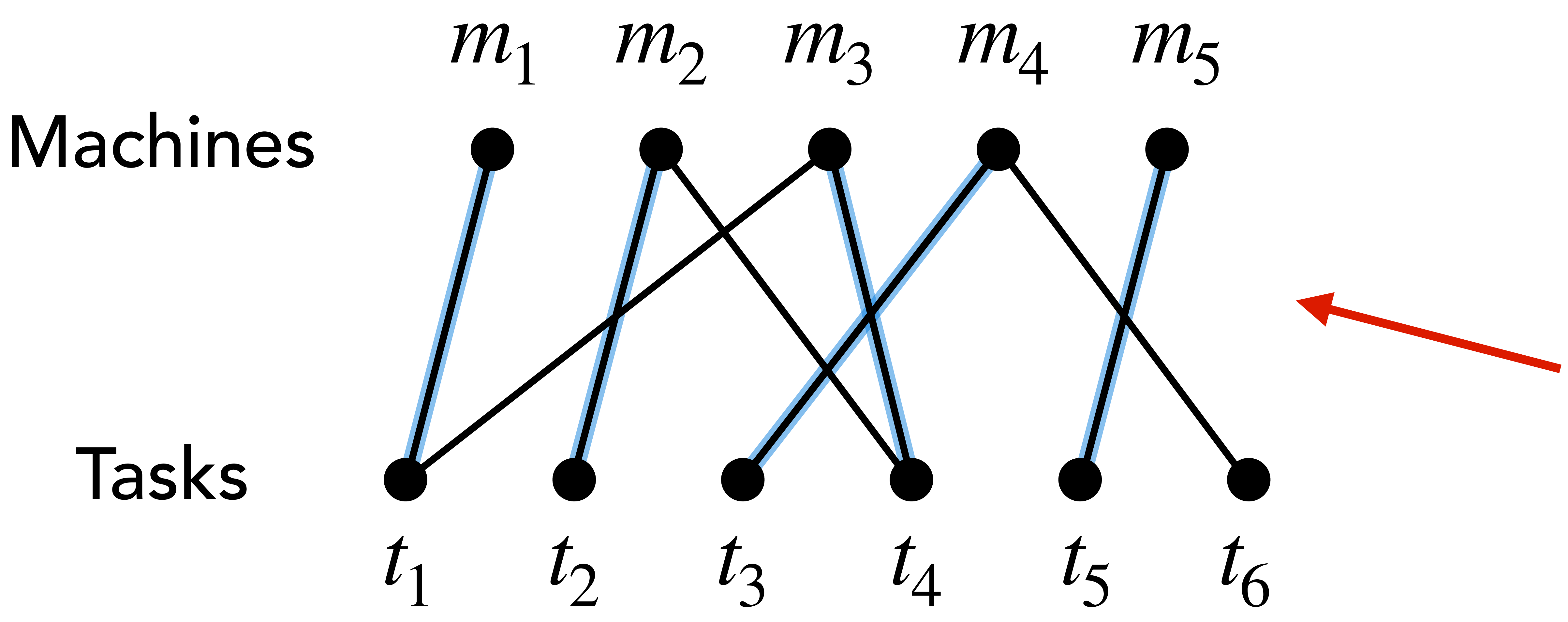

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Max-Bipartite-Matching:

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Input: A bipartite graph G.

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Has a surprising connection to *Max-flow*.

Bipartite Matching to Flow

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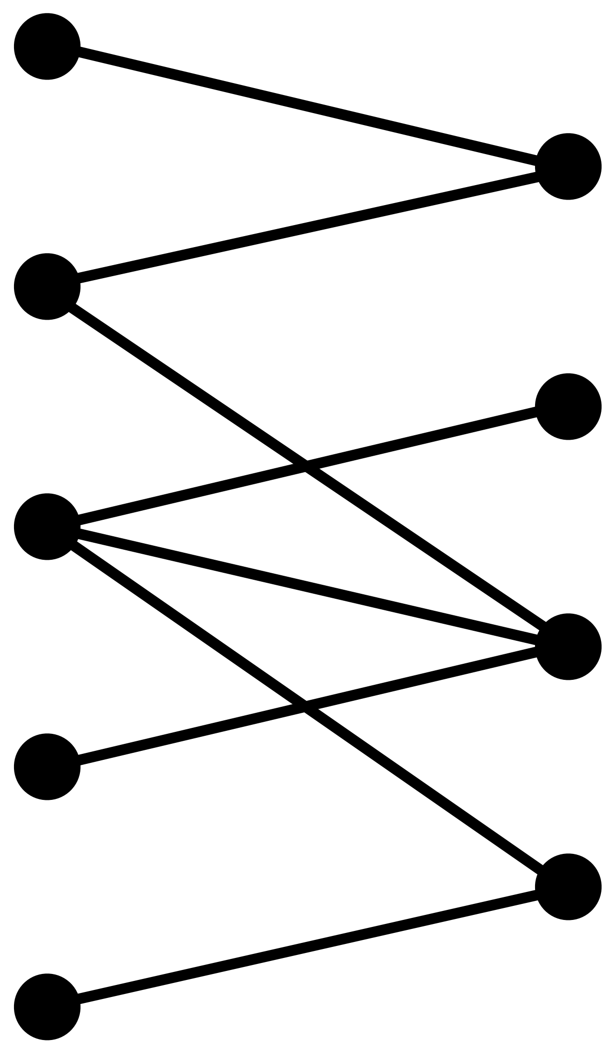

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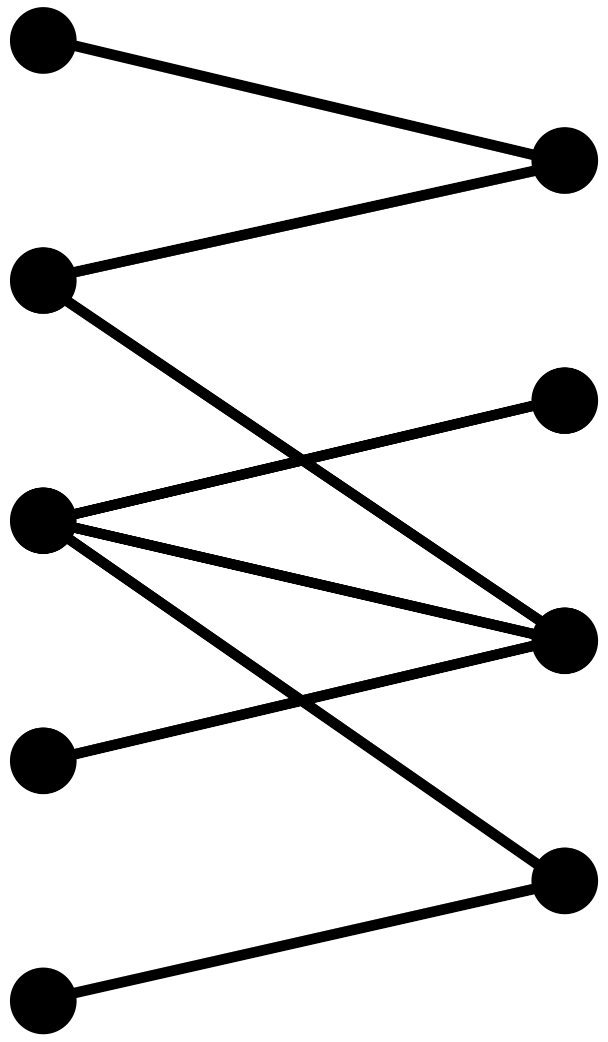

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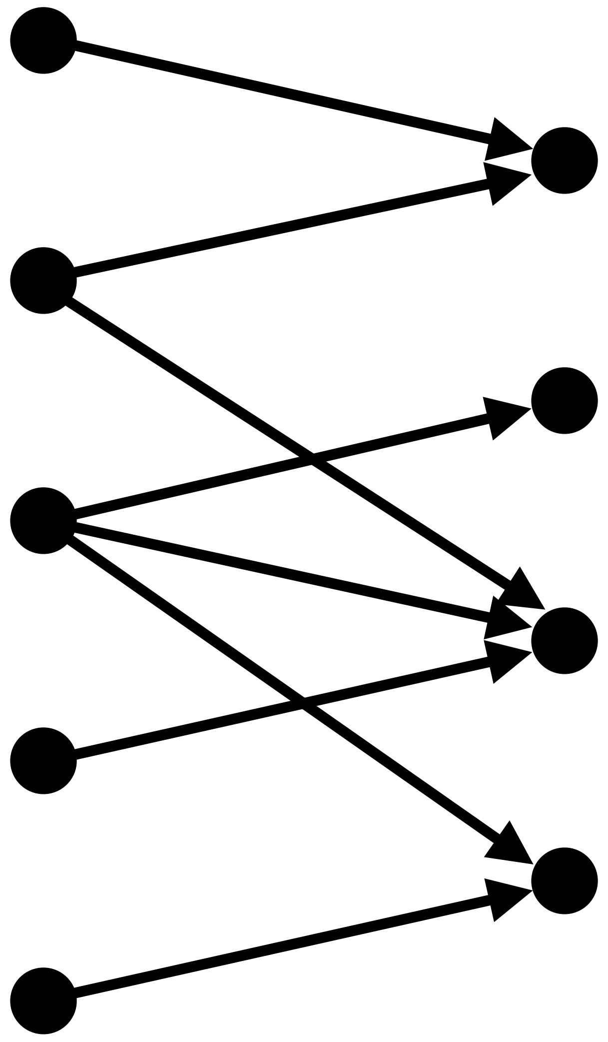

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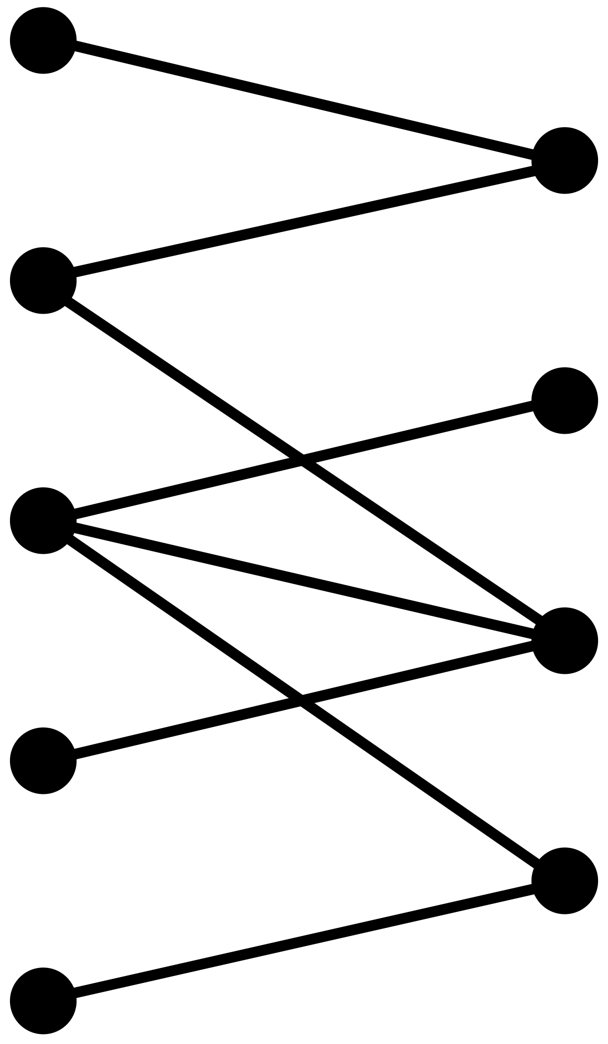

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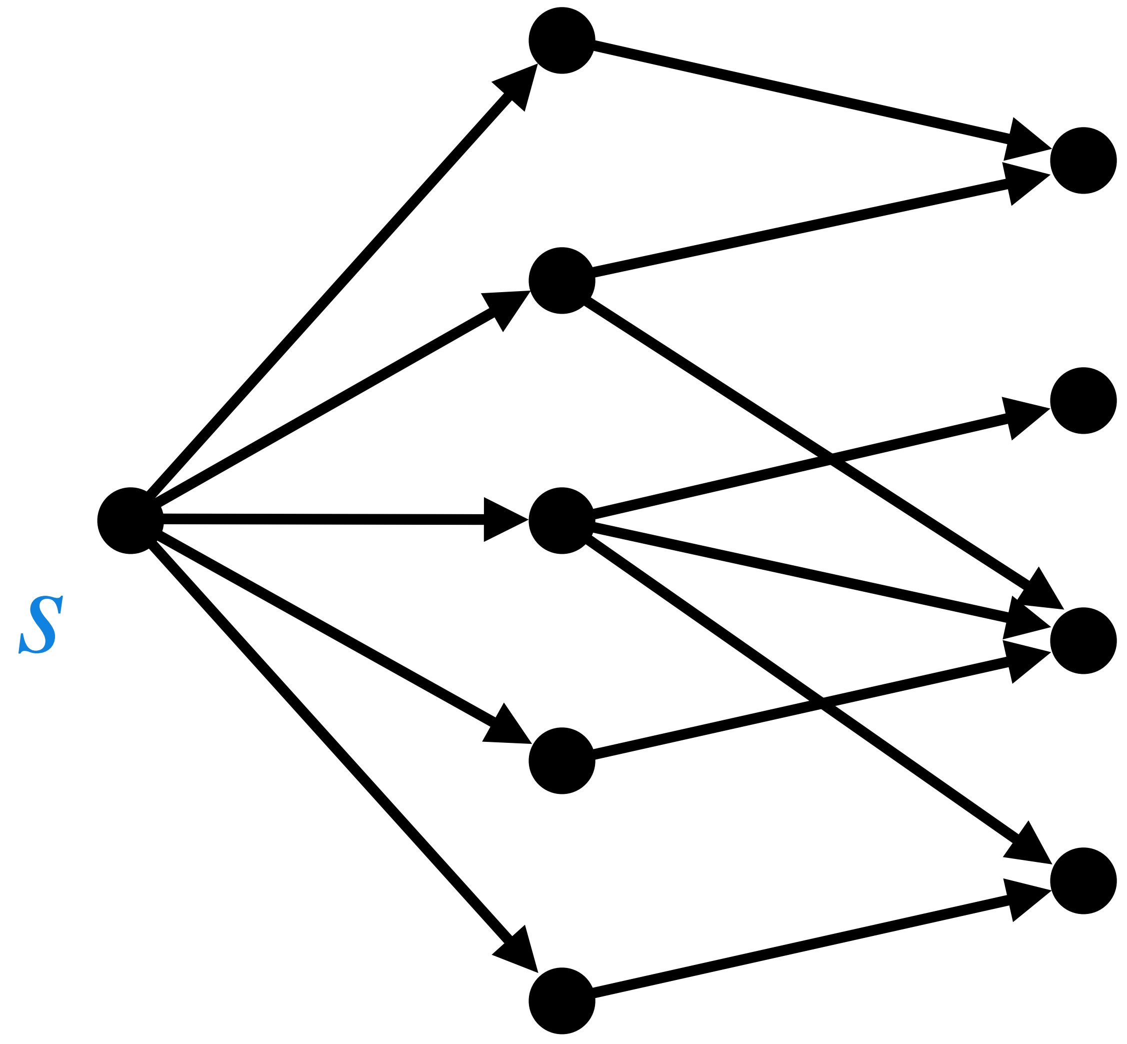

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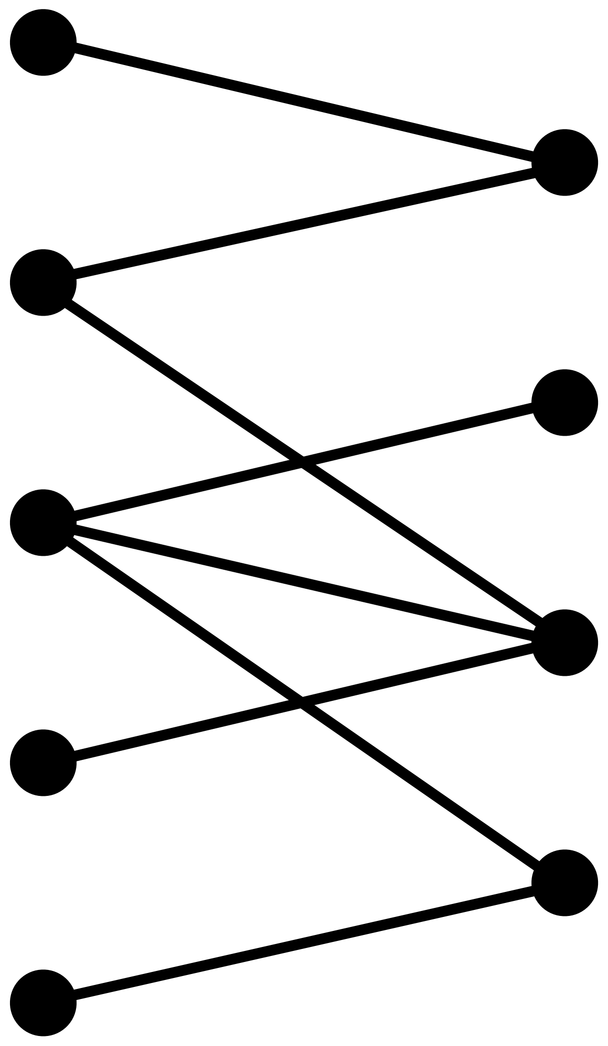

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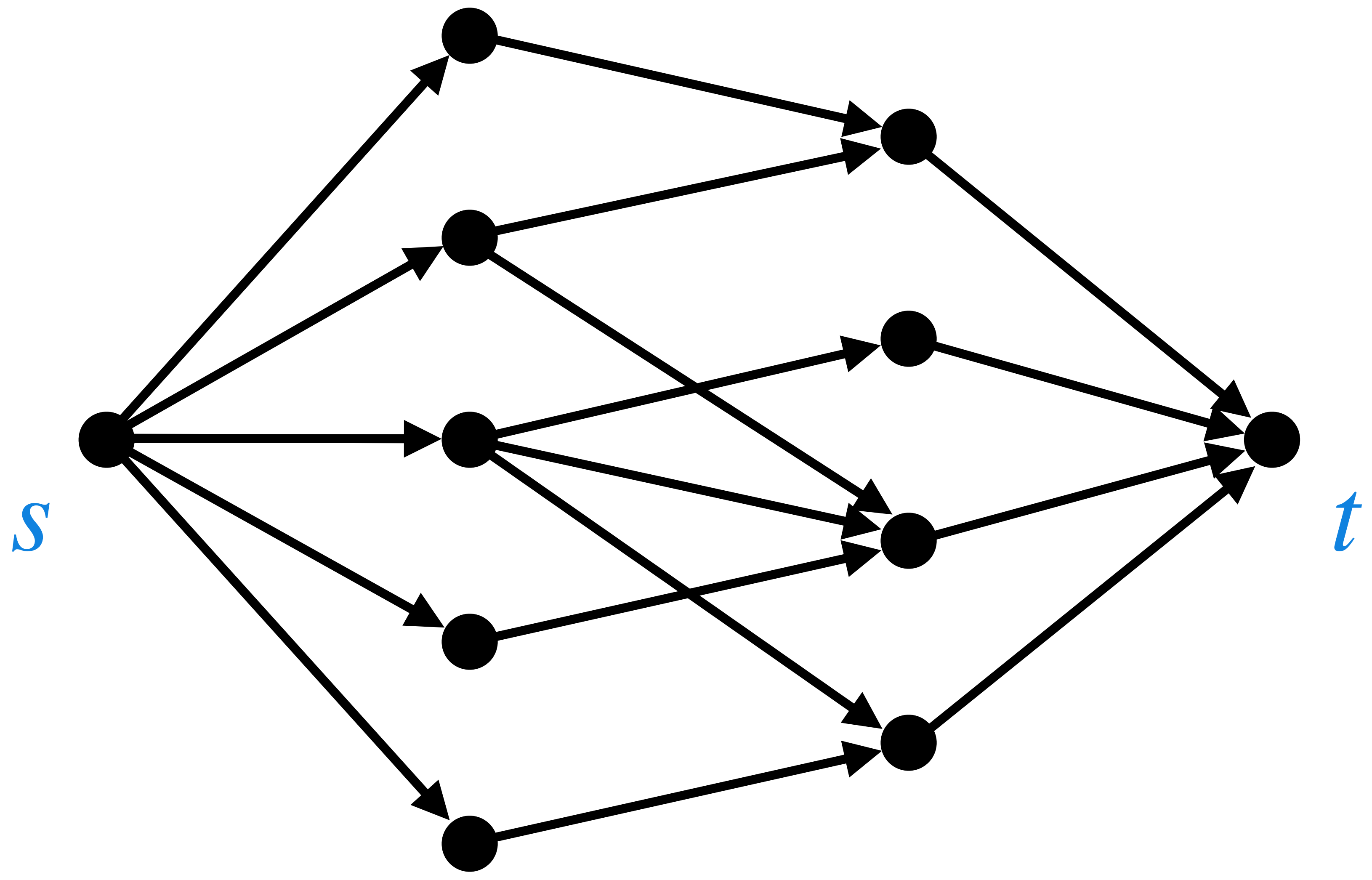

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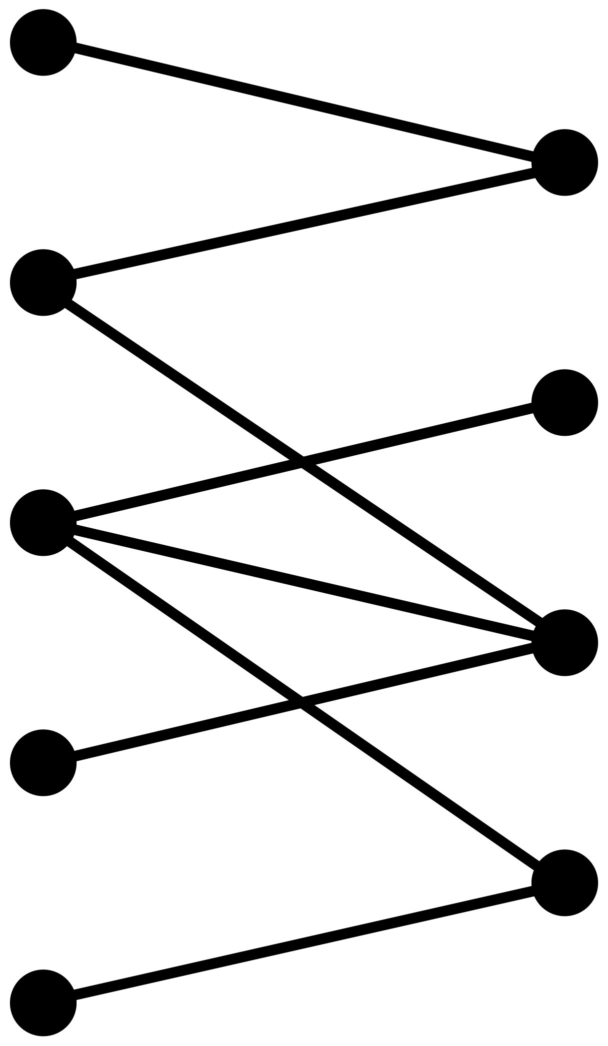

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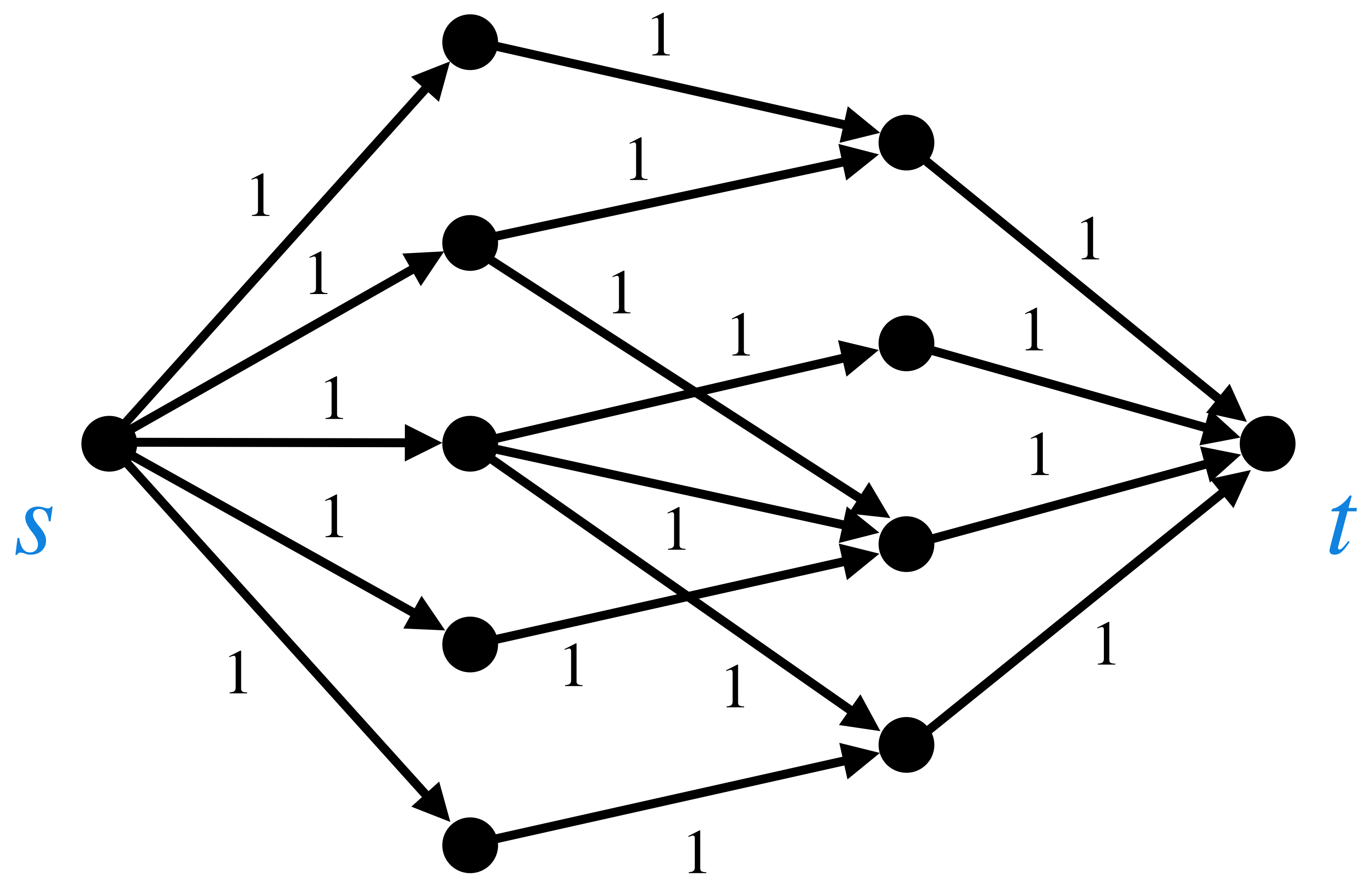

Bipartite Matching to Flow

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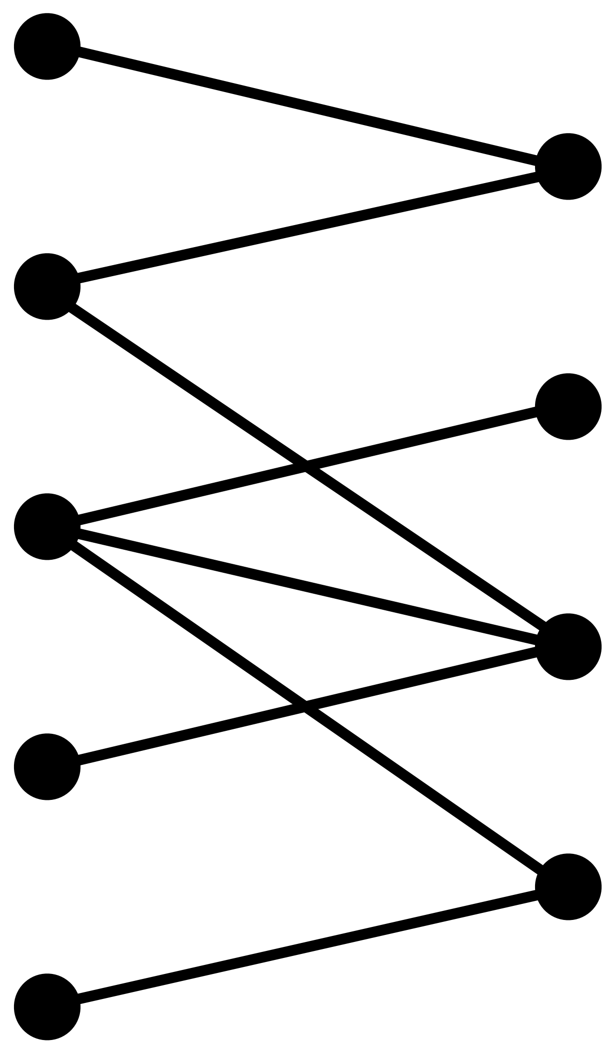

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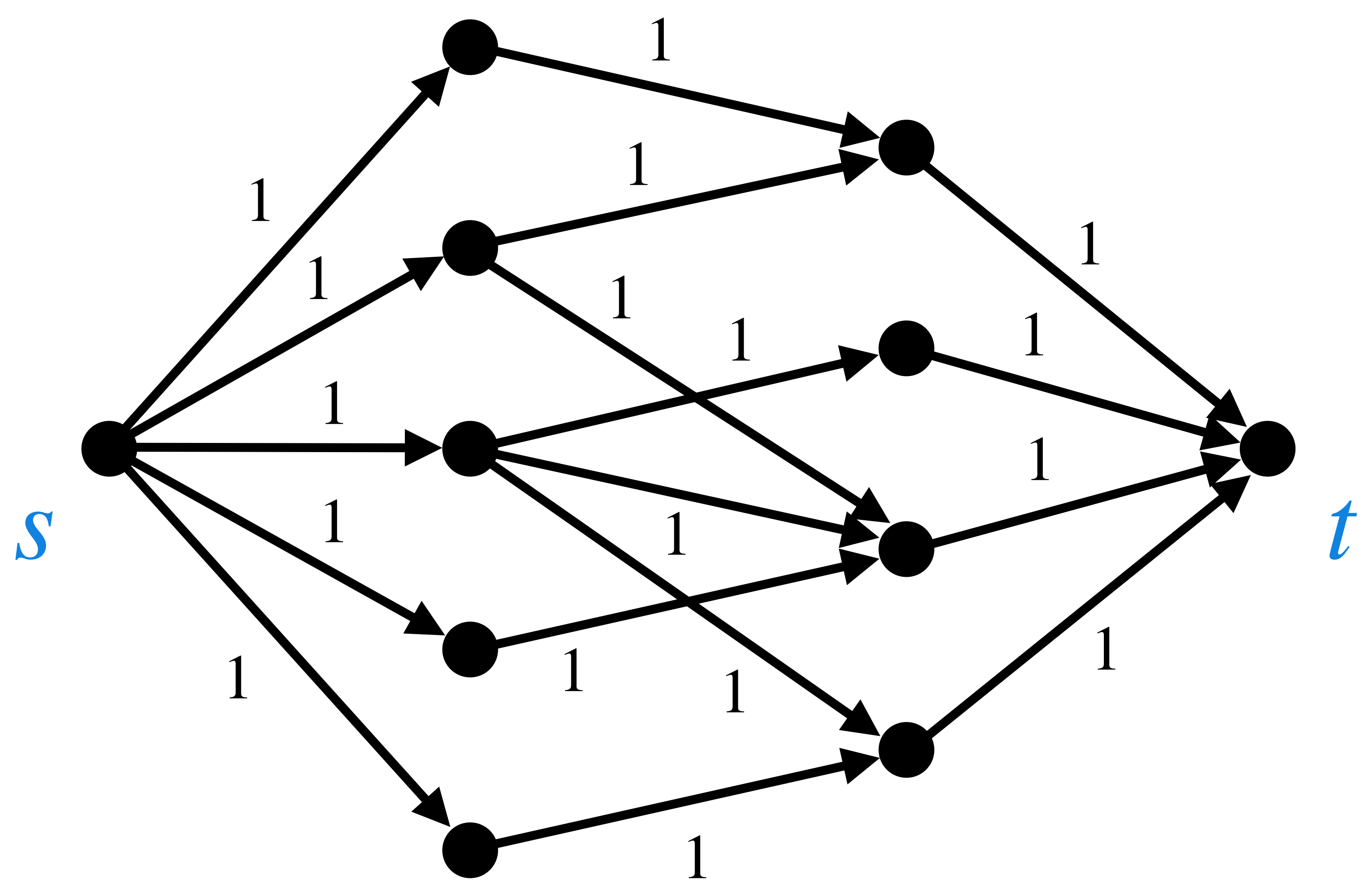

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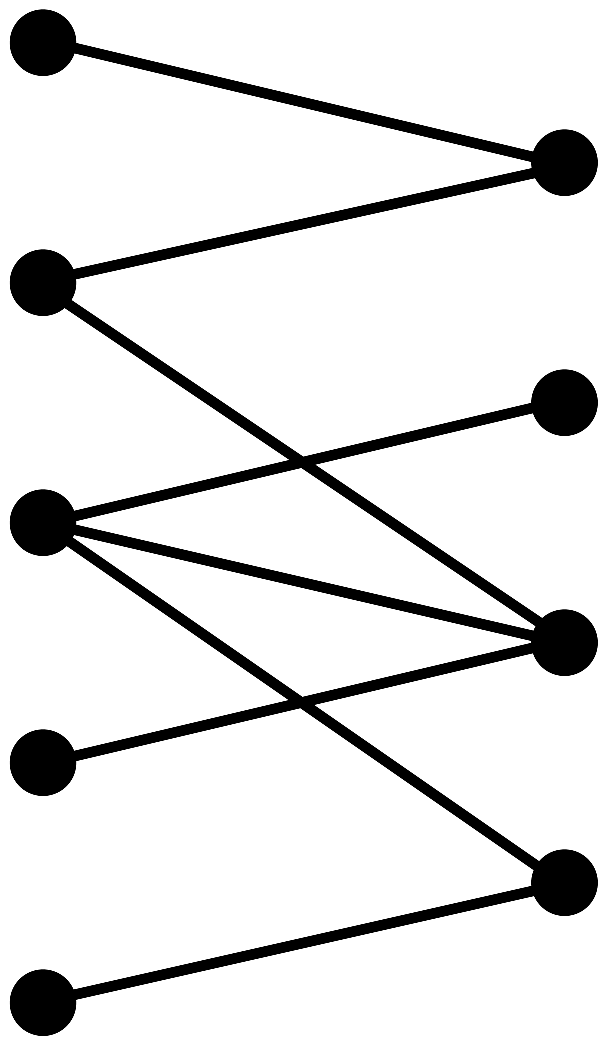

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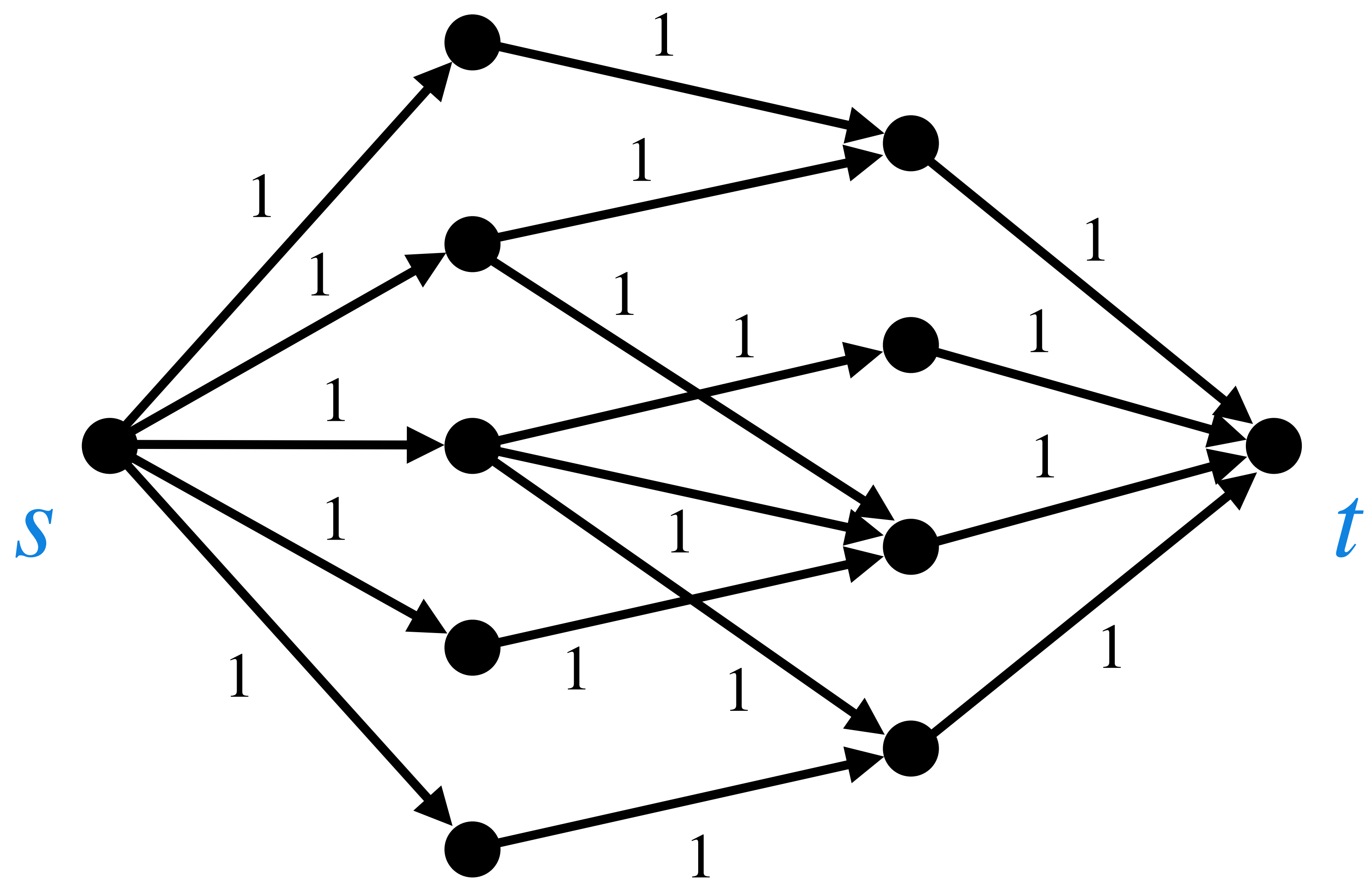

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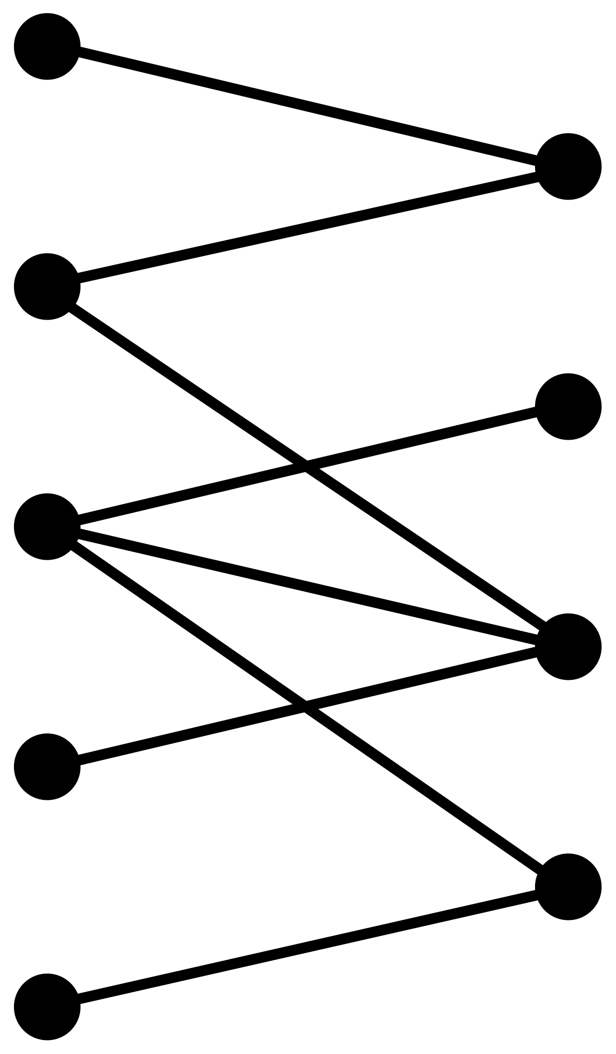

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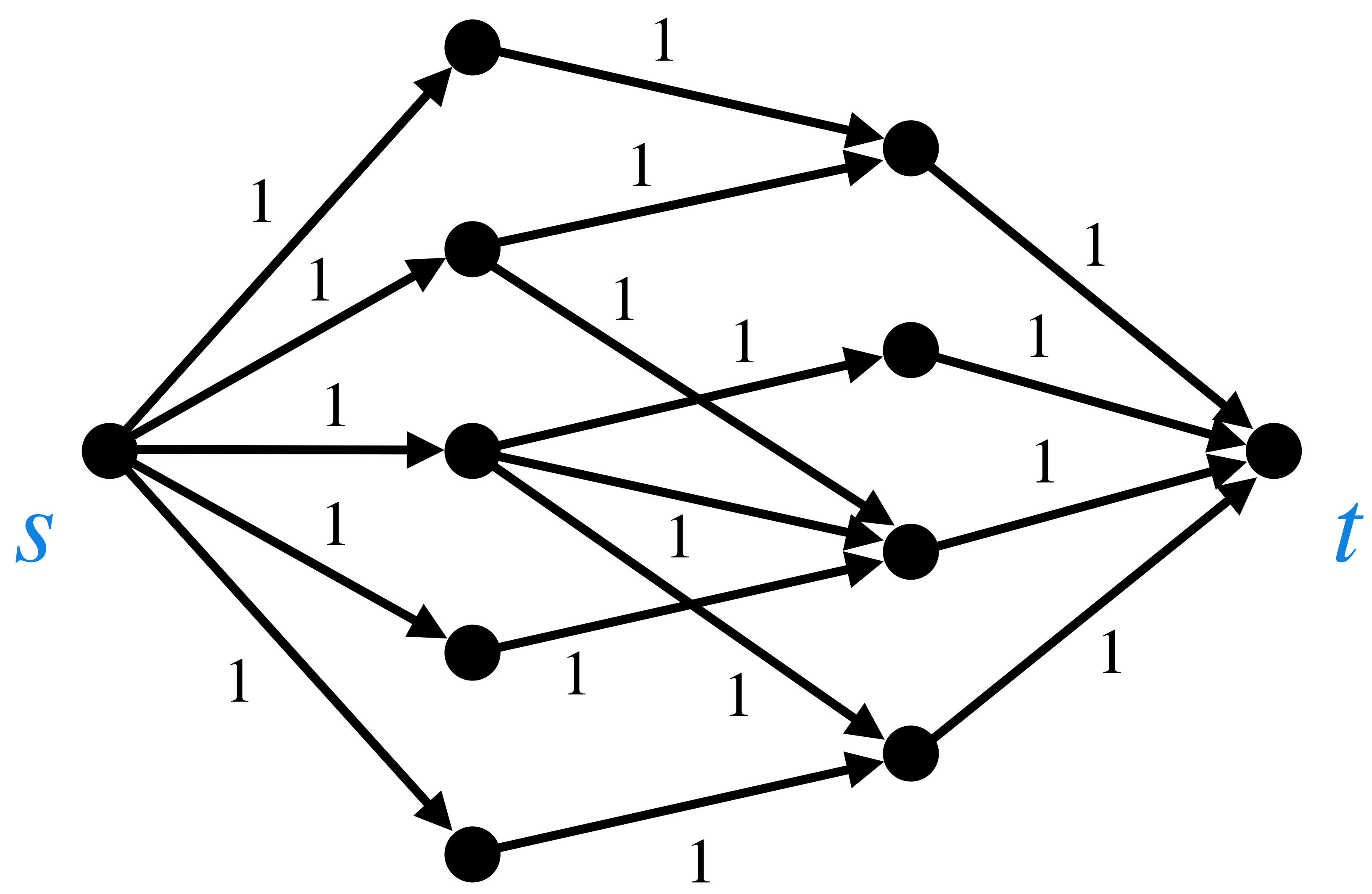

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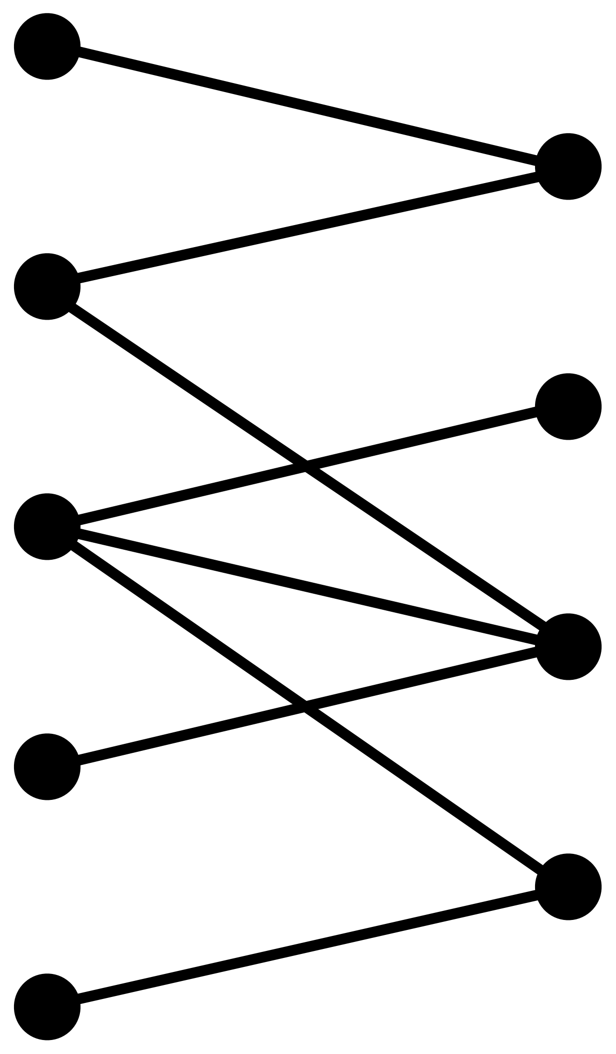

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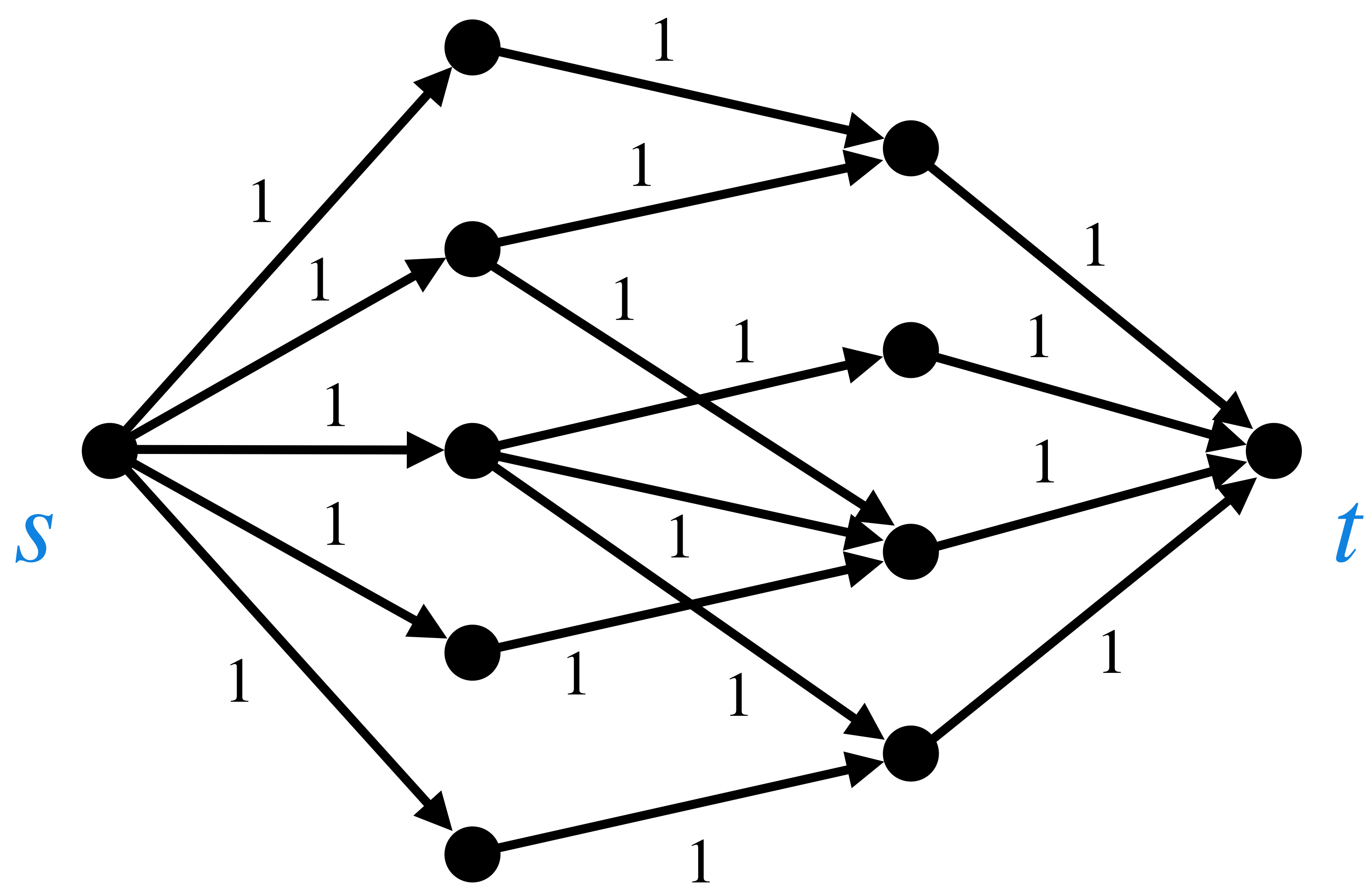

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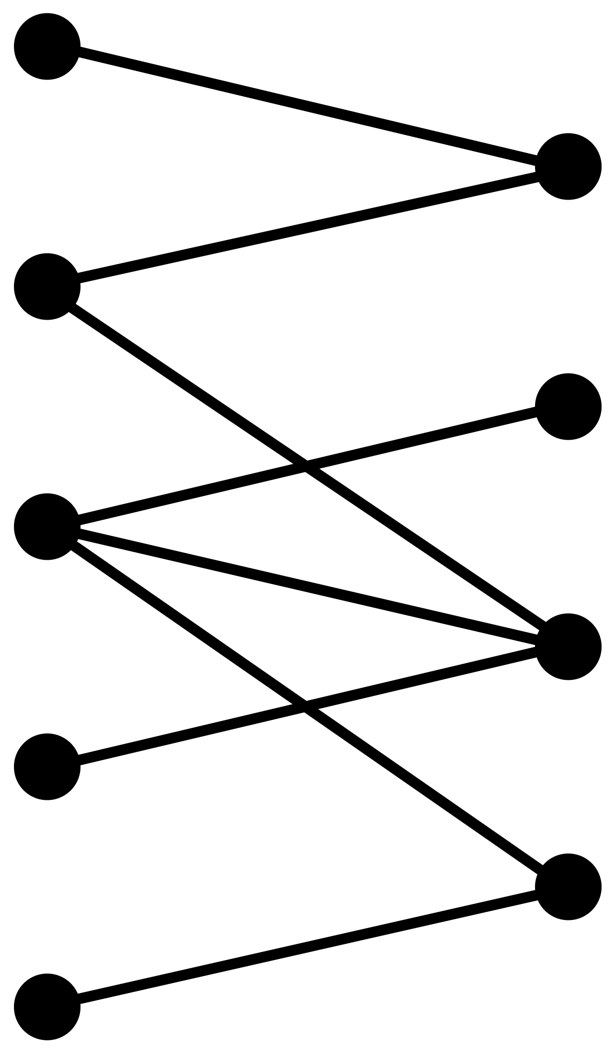

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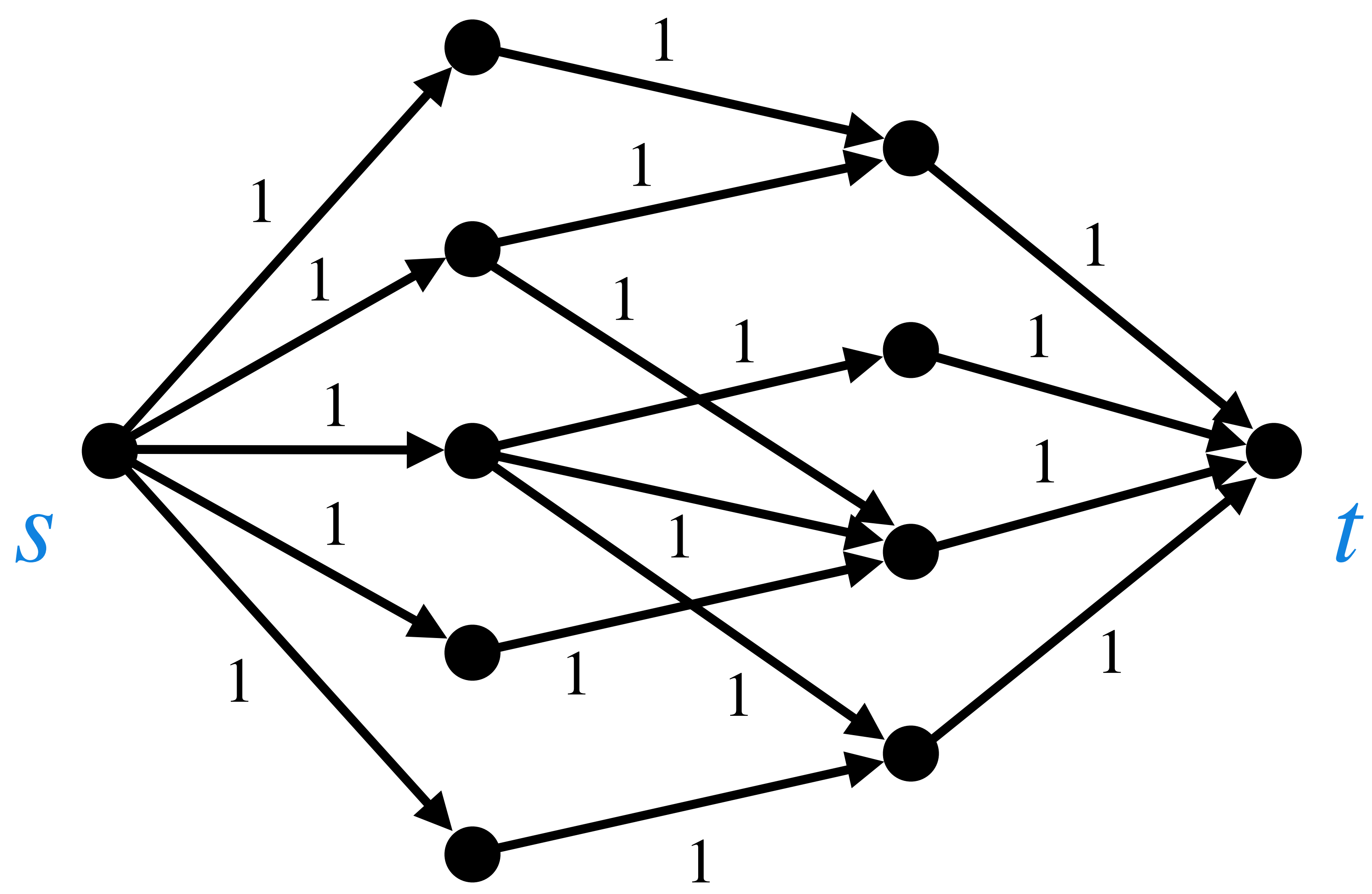

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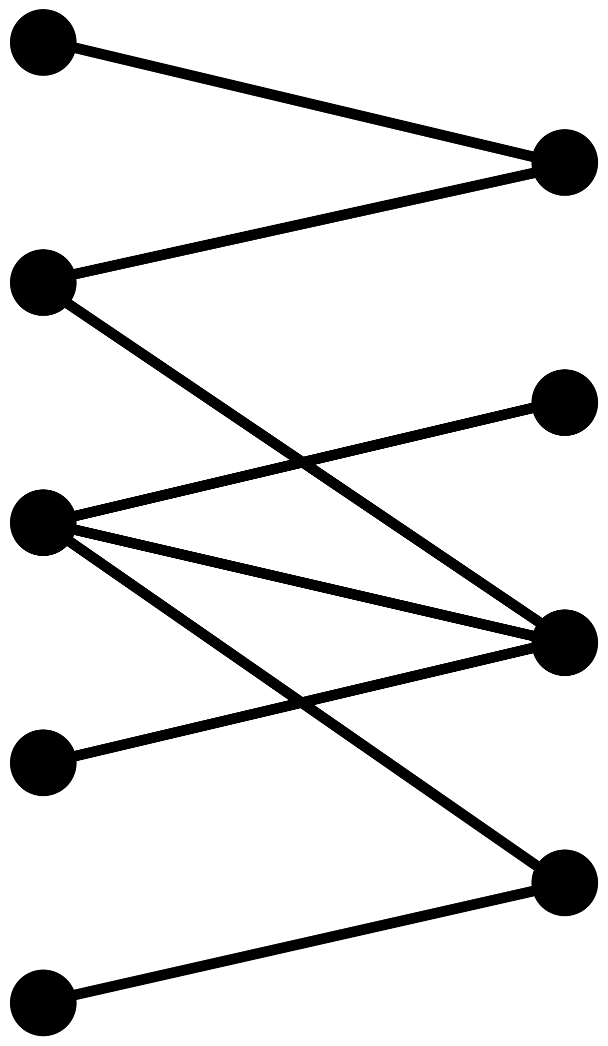

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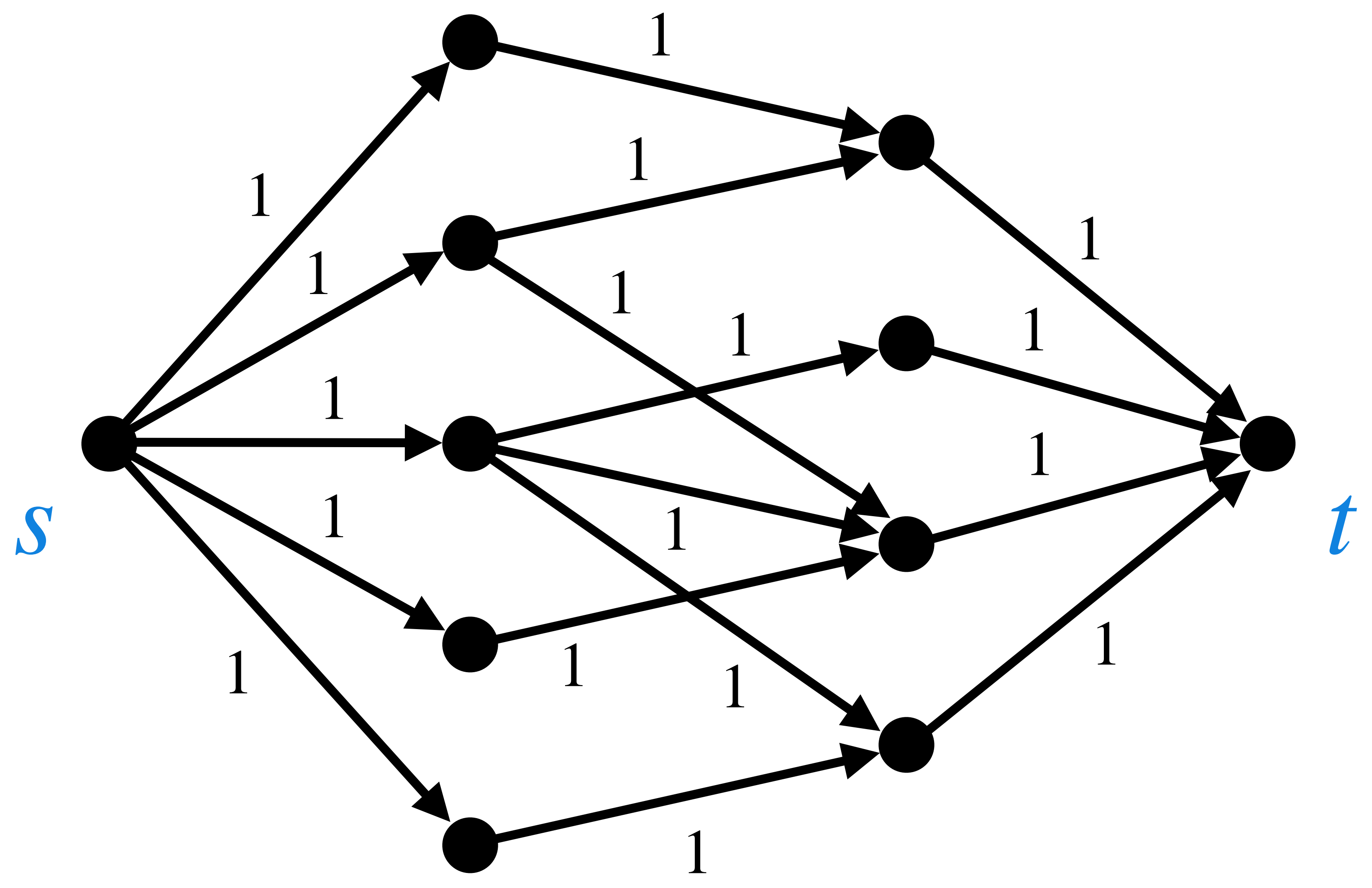

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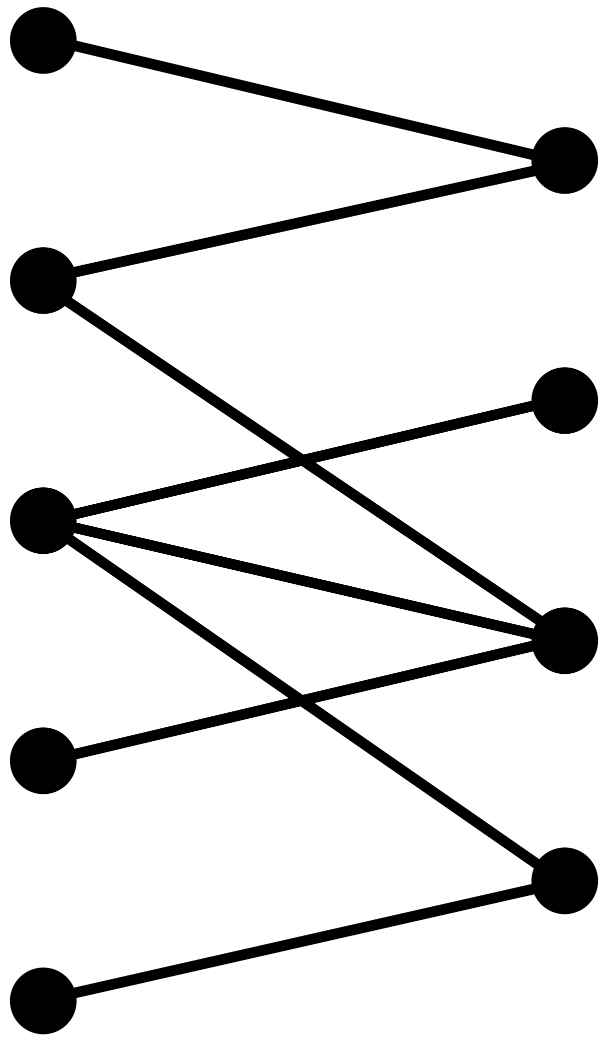

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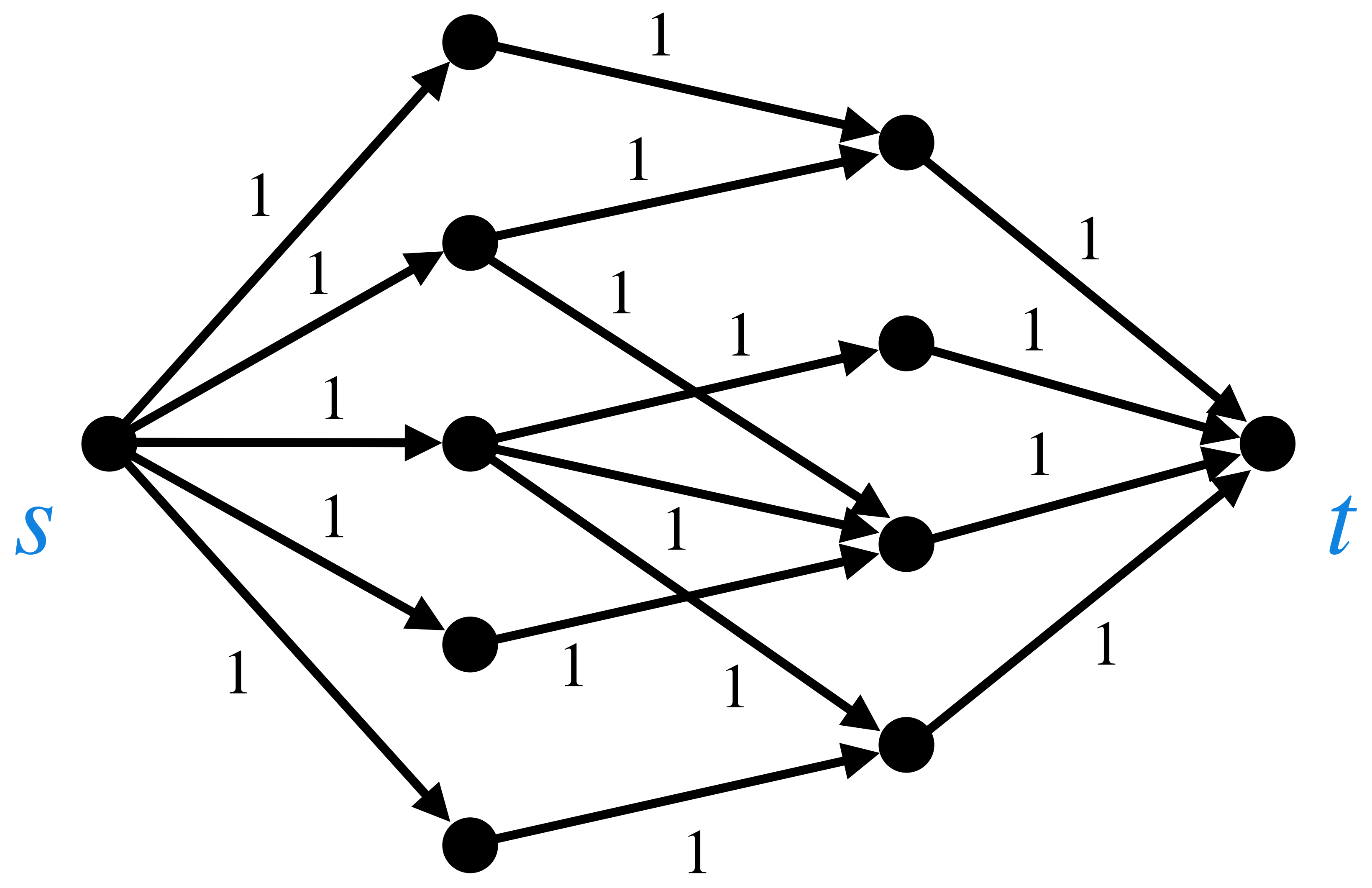

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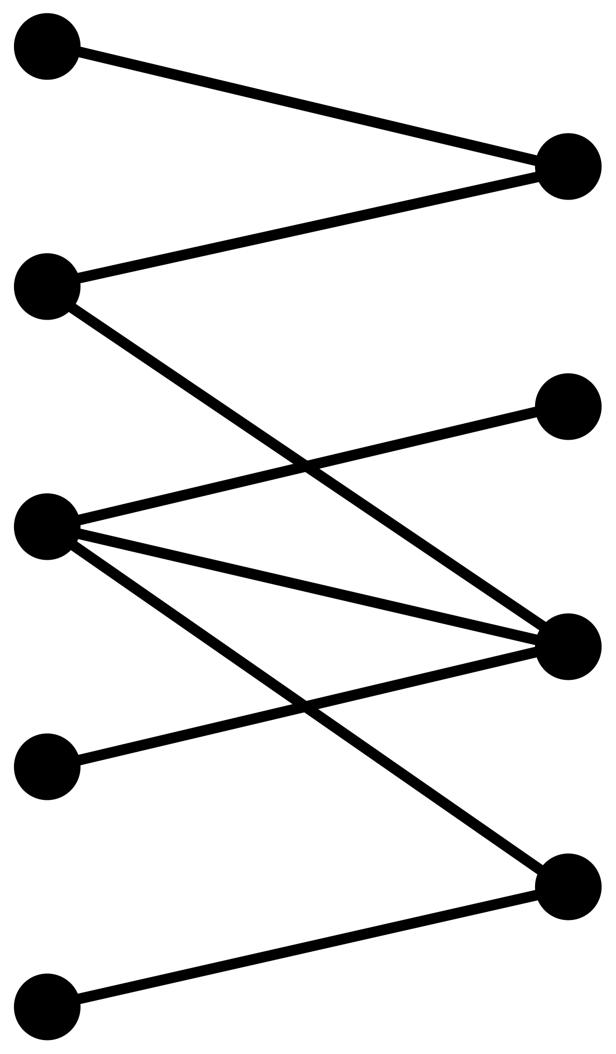

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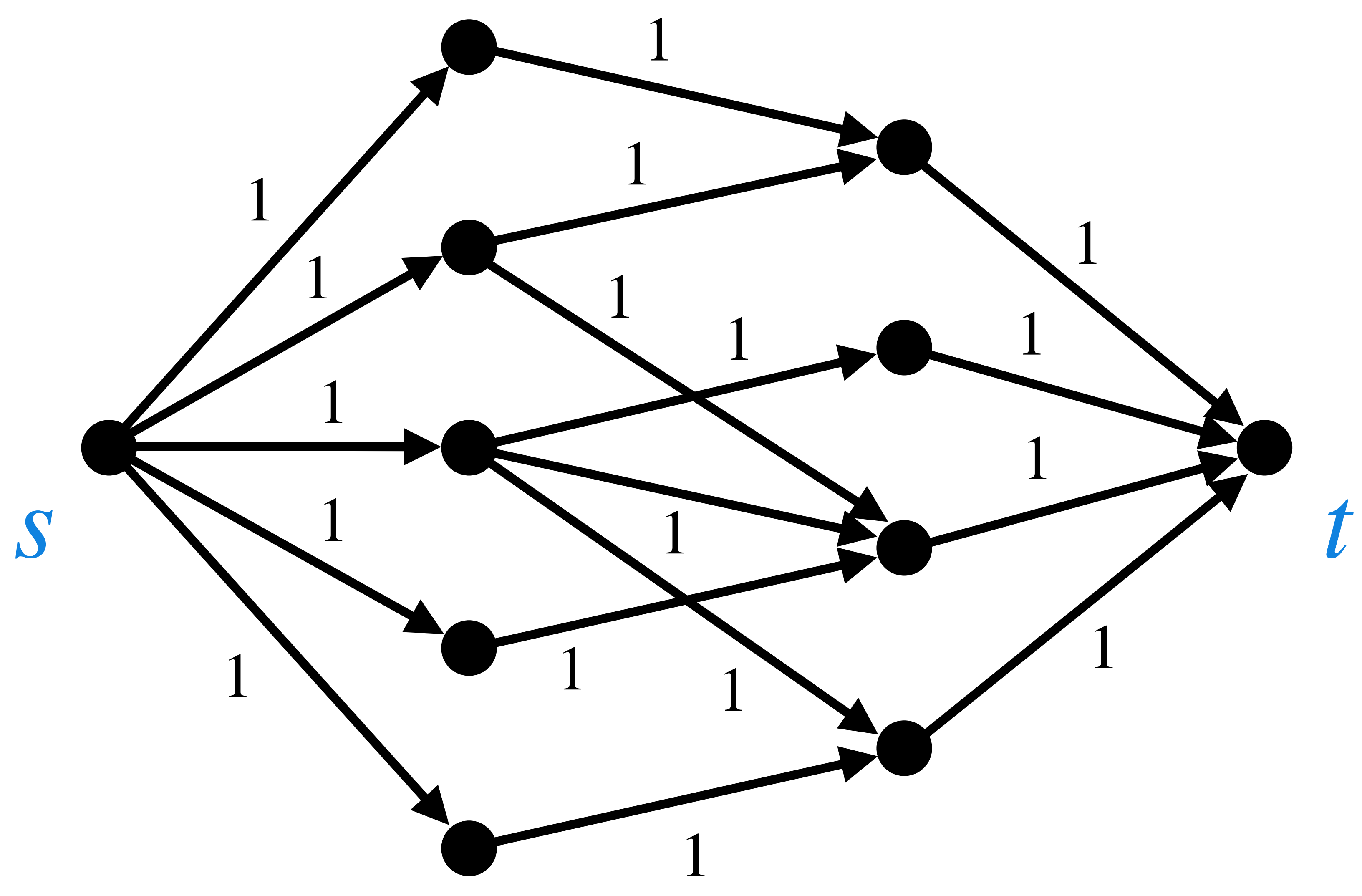

Bipartite Matching to Flow

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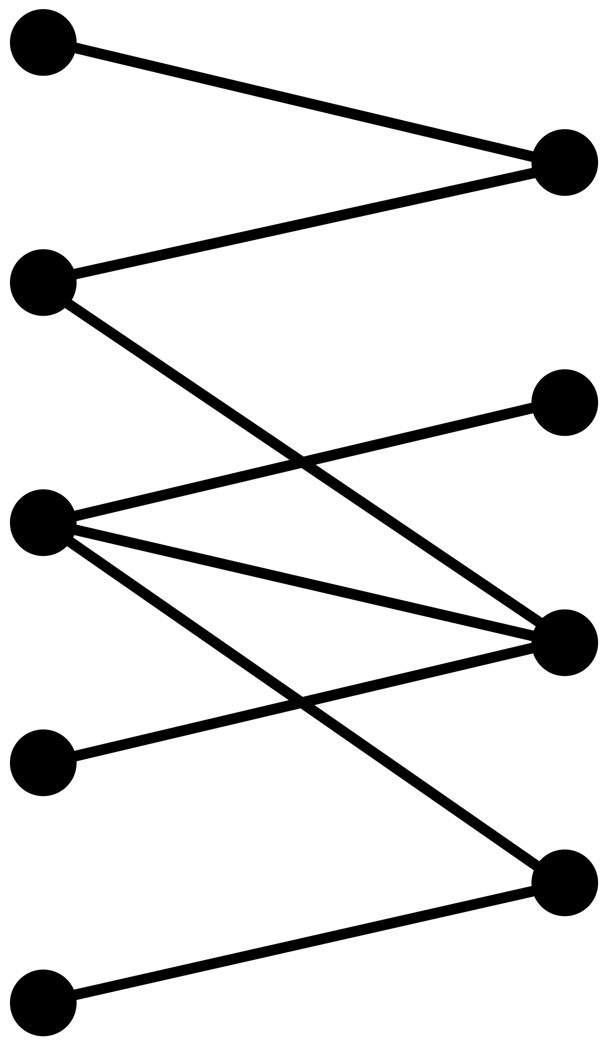
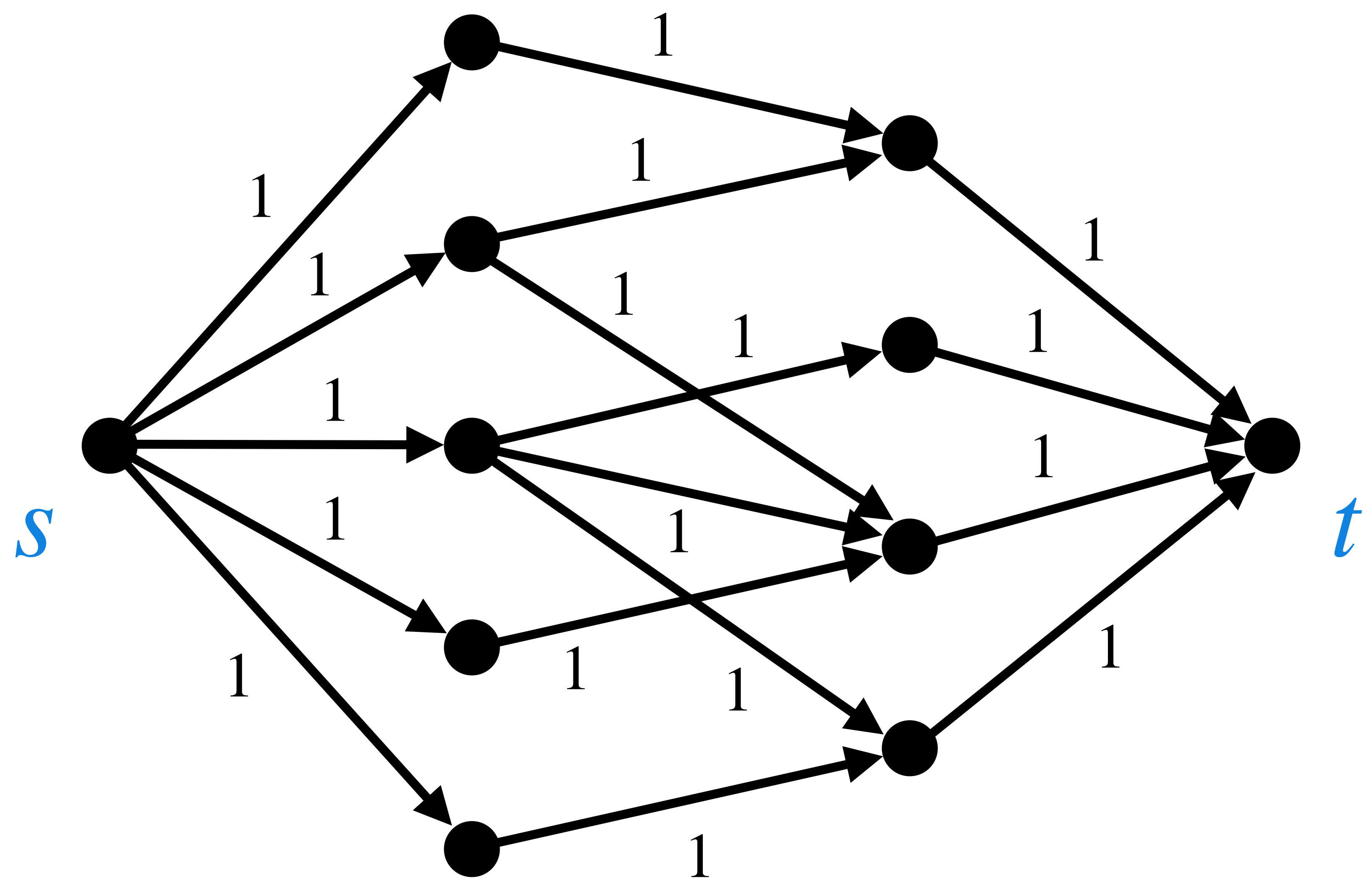


$G' = (V', E')$



Bipartite Matching to Flow

Goal: We want a way to compute maximum matching in G by computing max-flow in G'.

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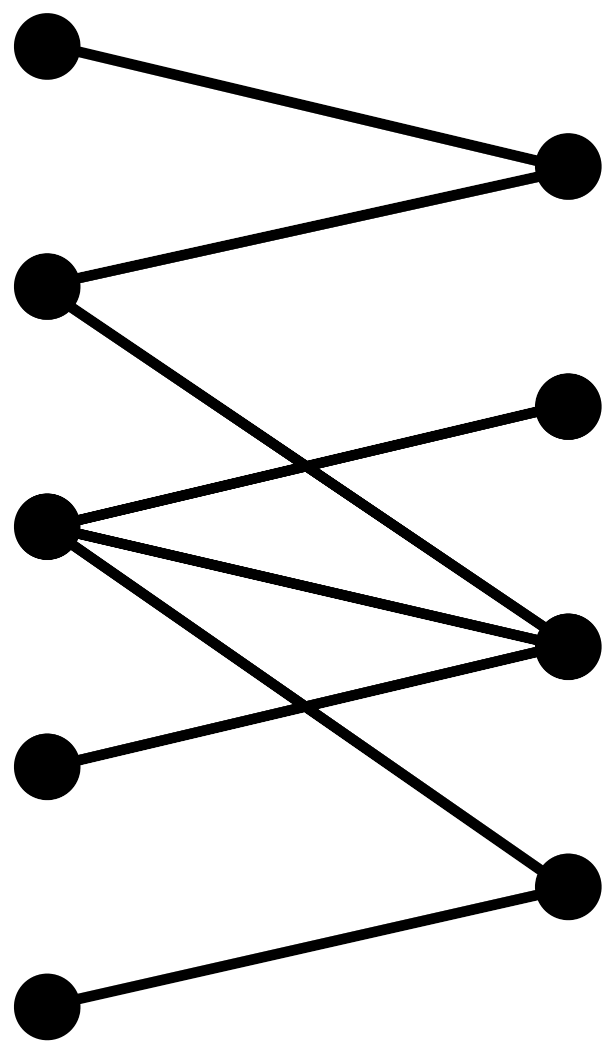

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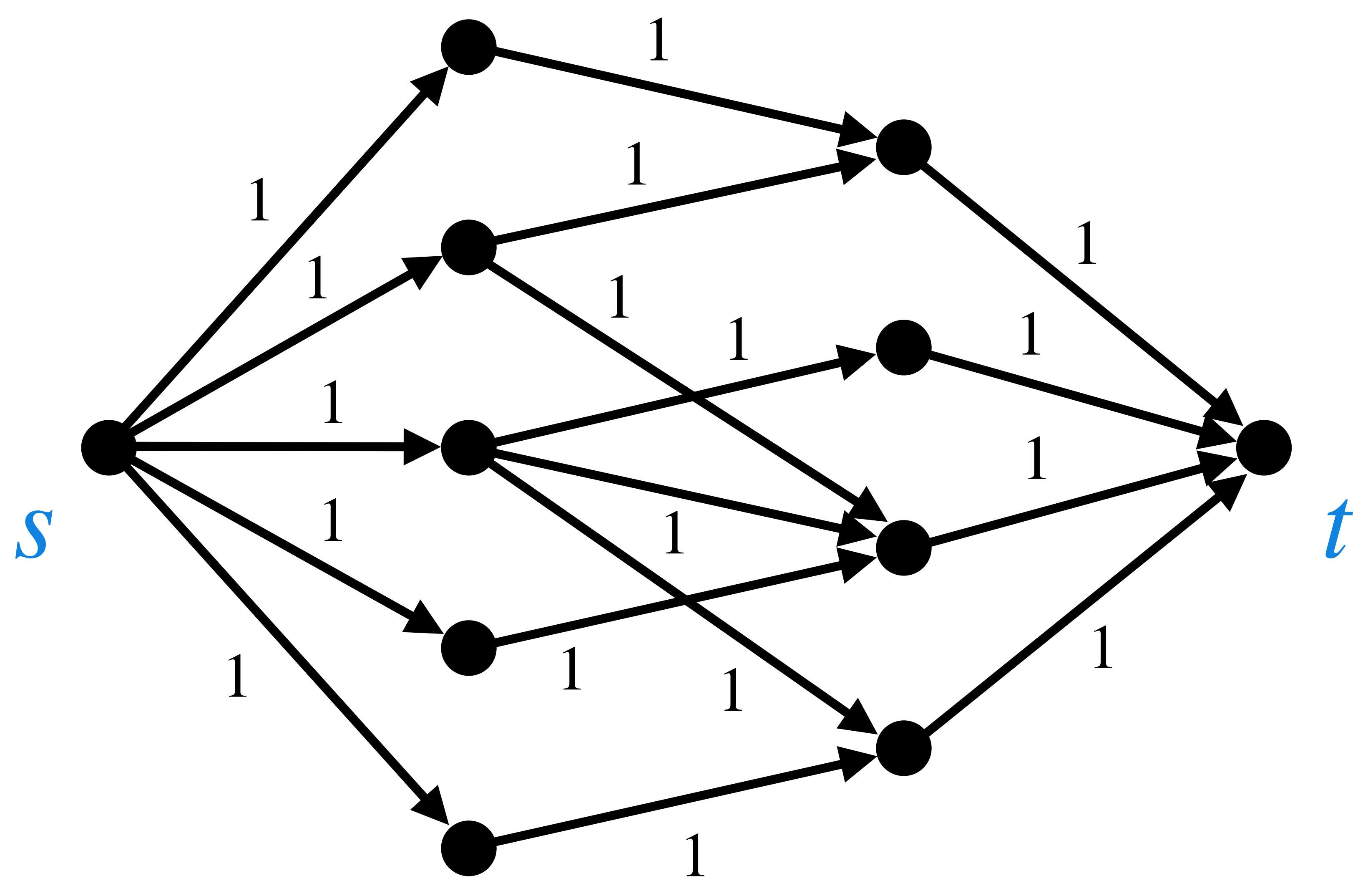

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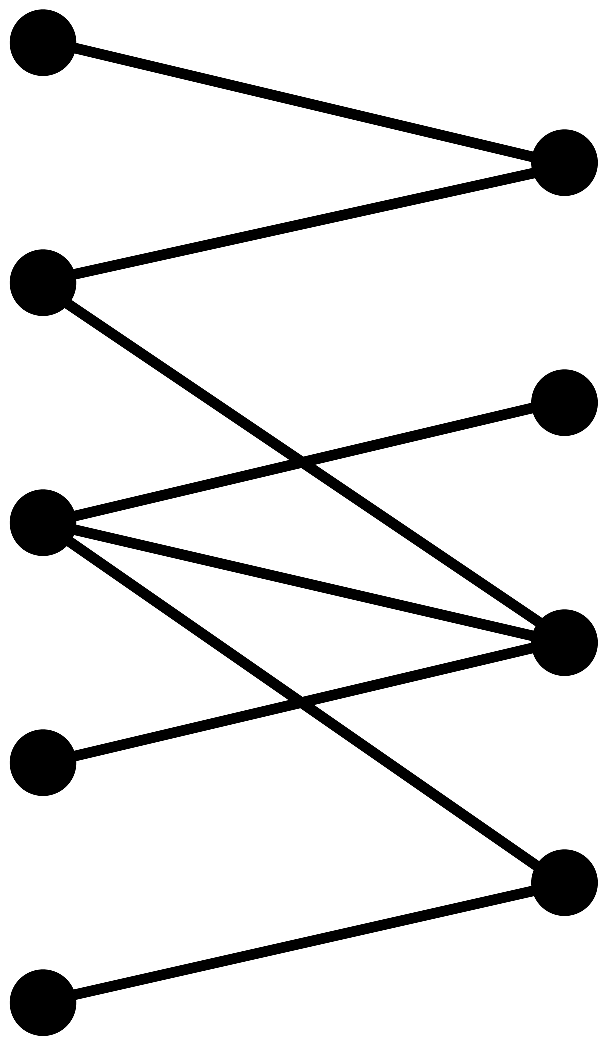

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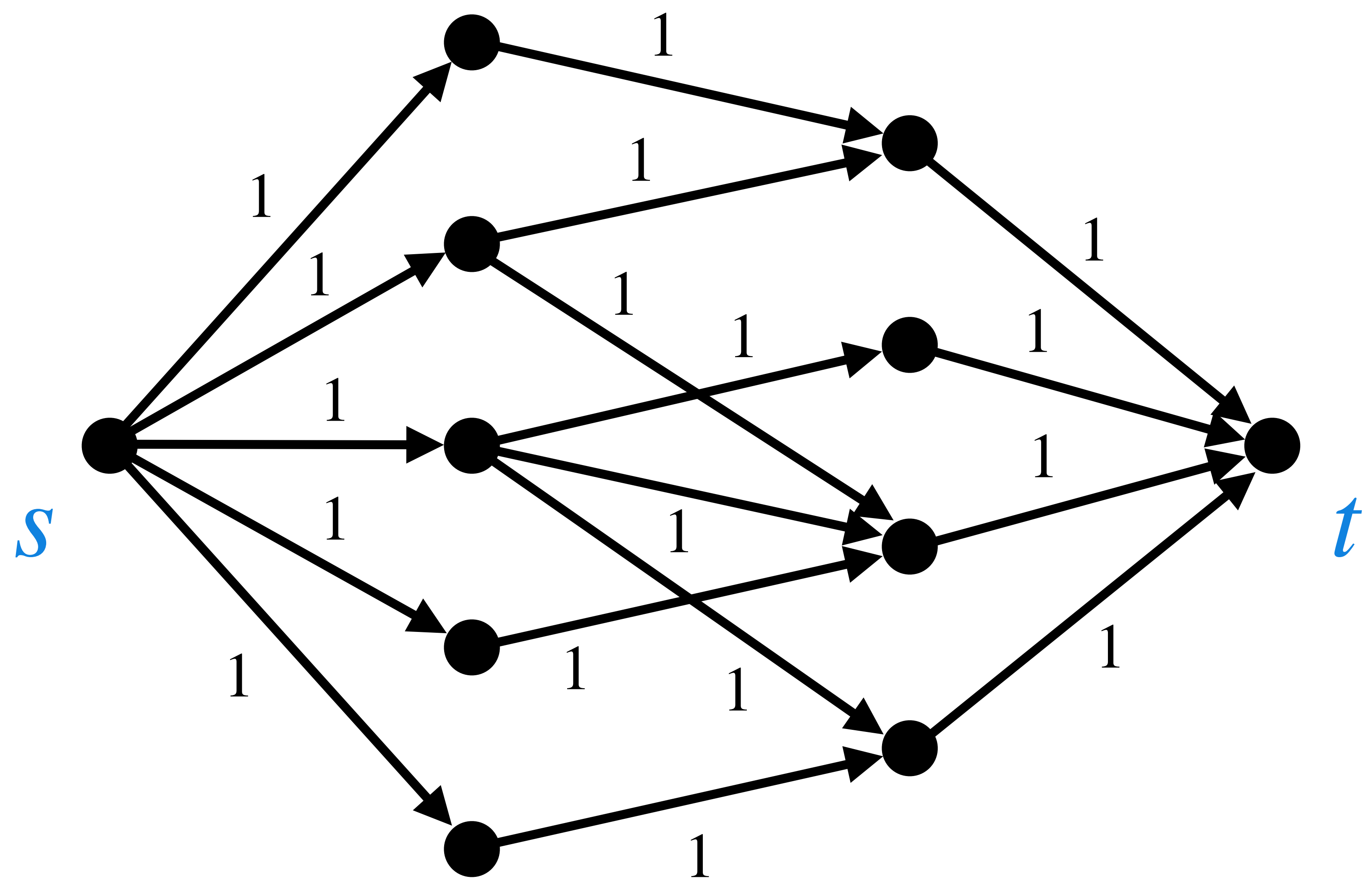

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Claim: If M is a matching in G, then there is an **integer-valued** f in G' with value $|f| = |M|$.

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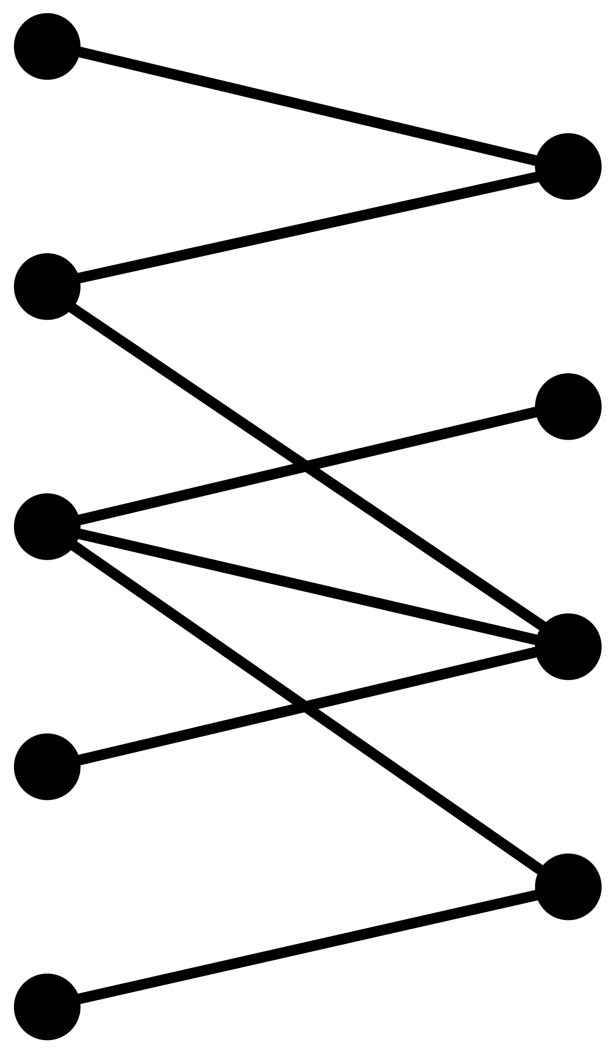

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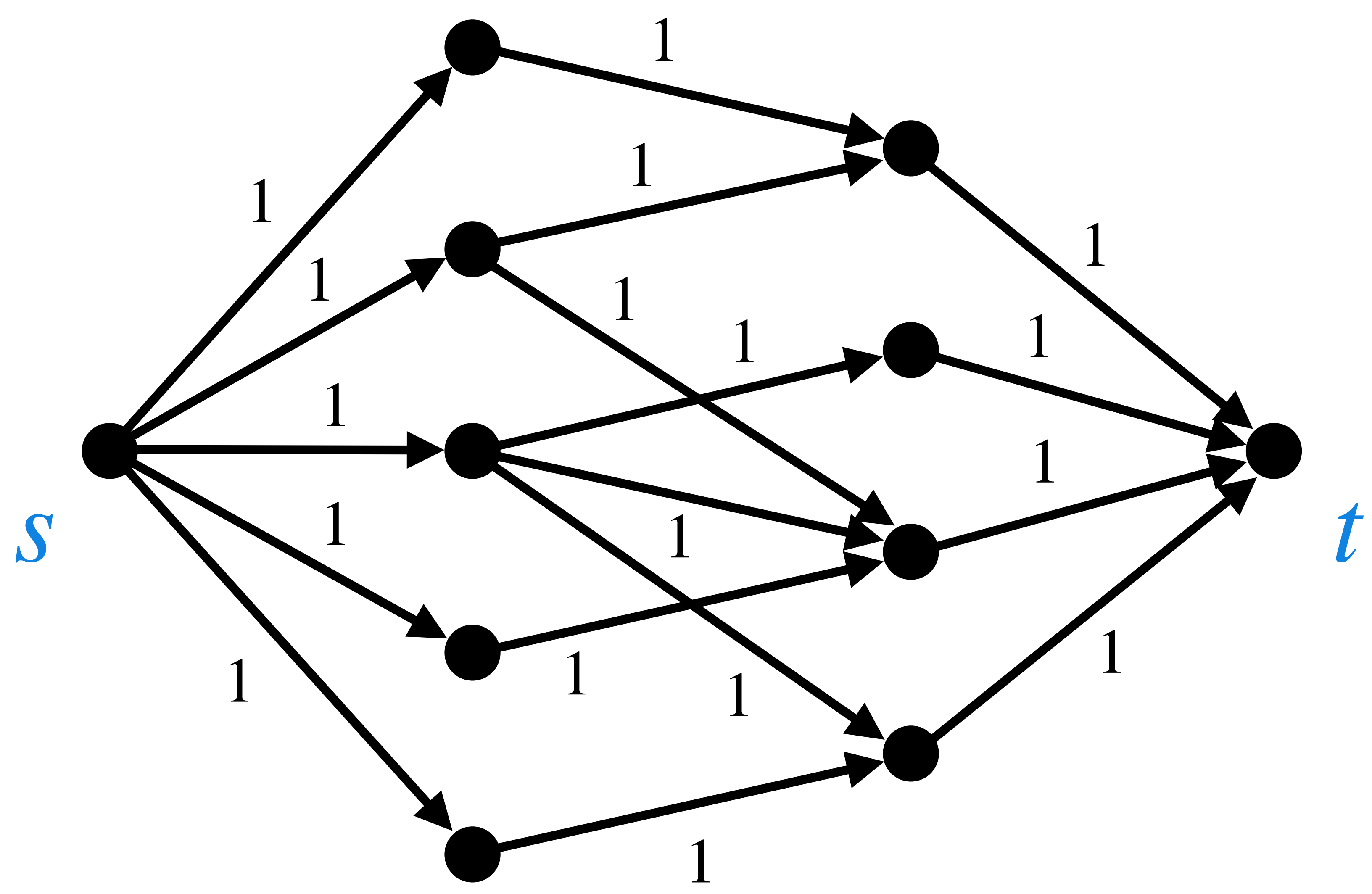

Bipartite Matching to Flow

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Flow where $f(u, v)$ is an integer for every (u, v).

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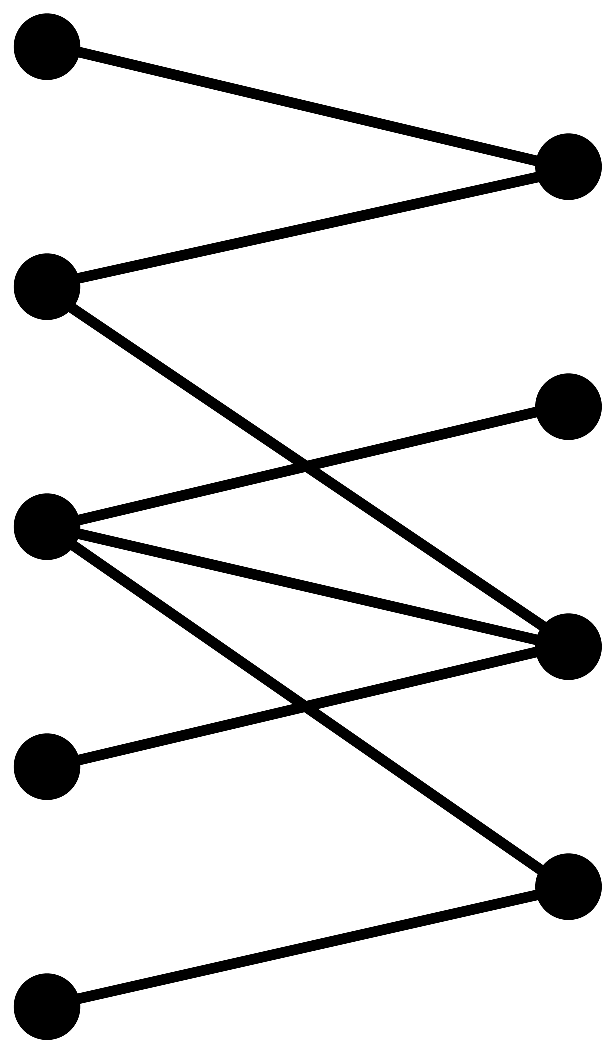

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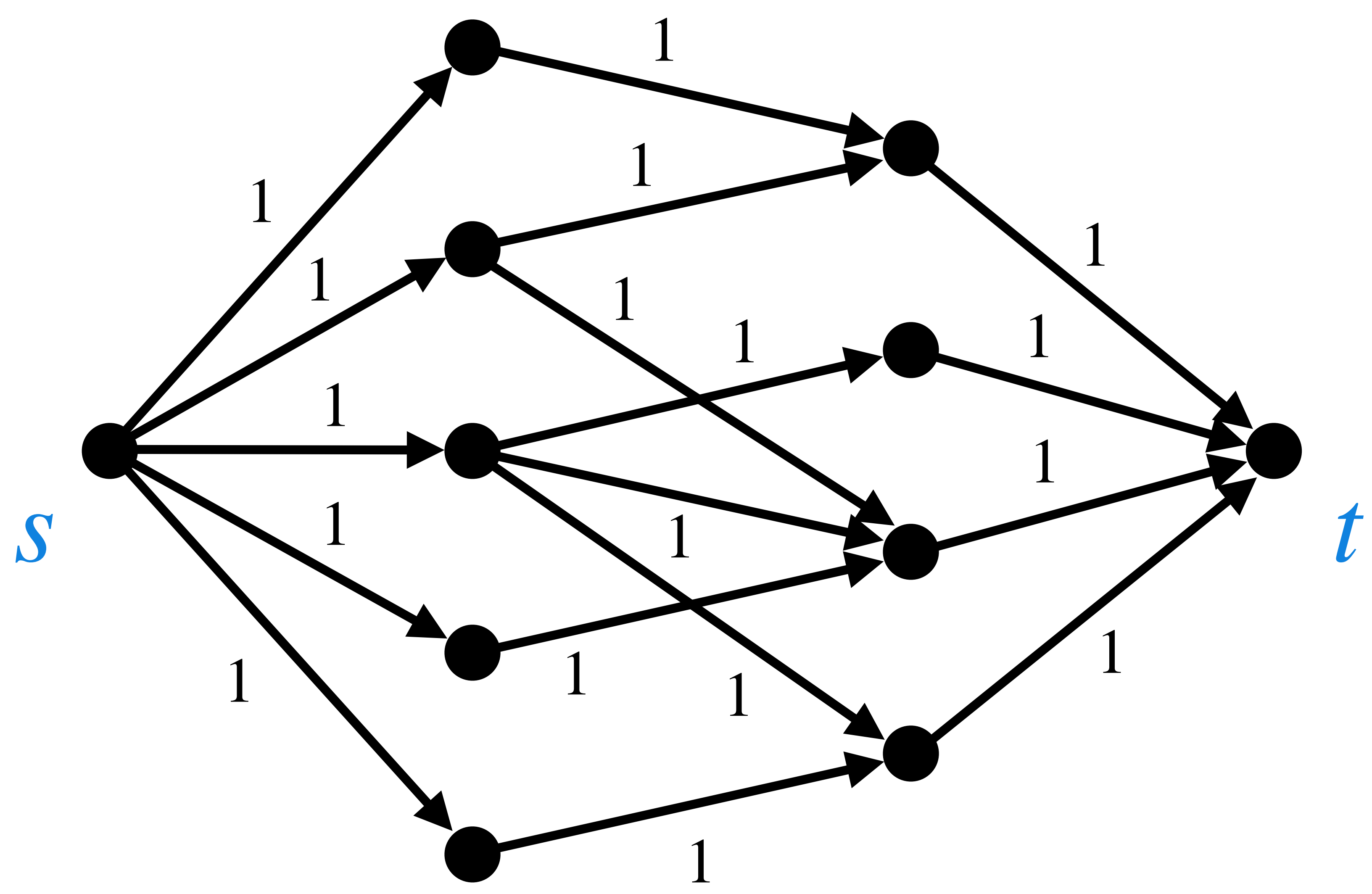

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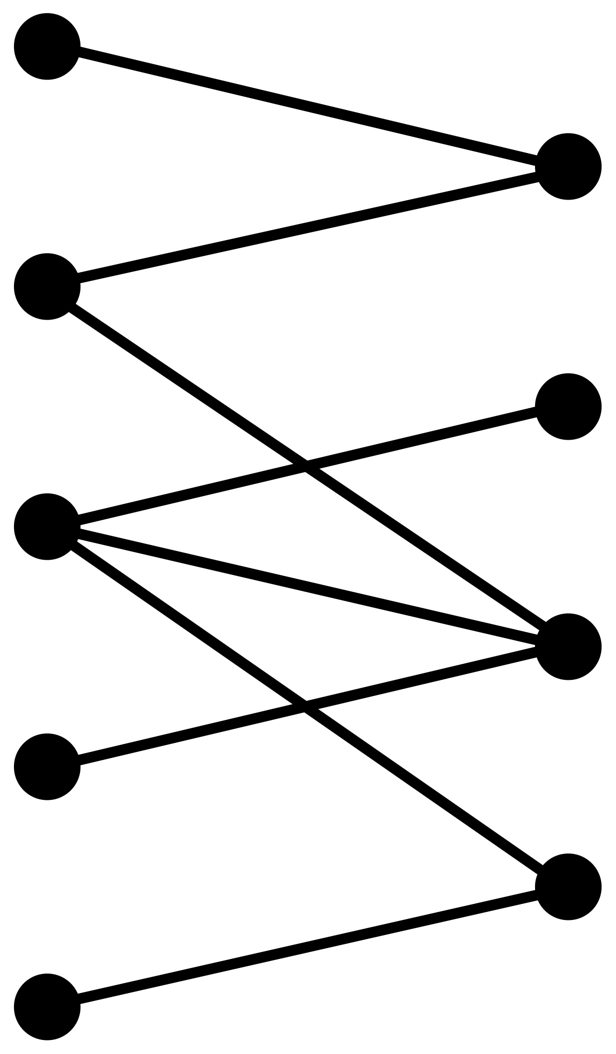

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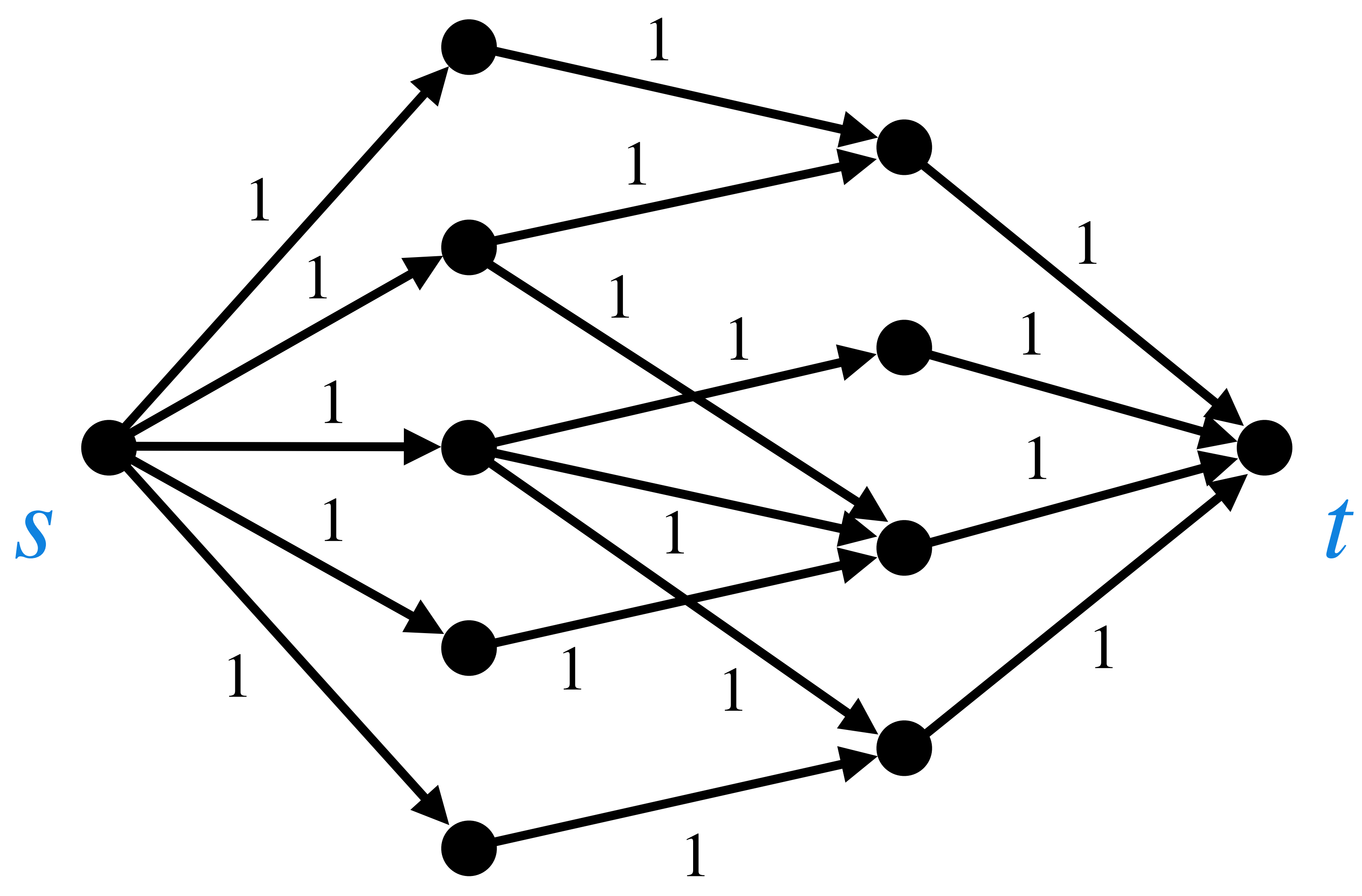

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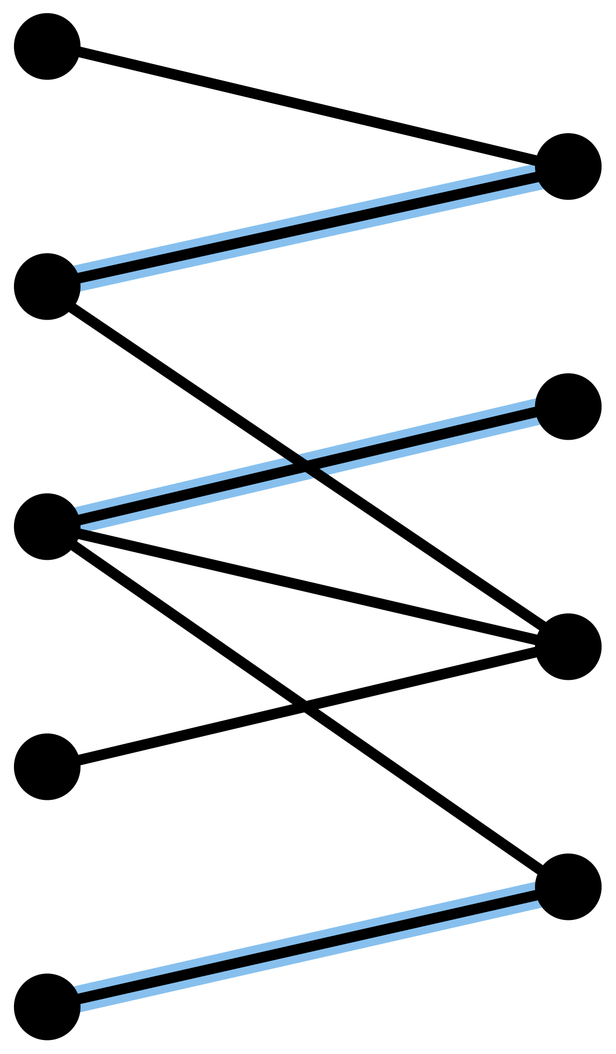

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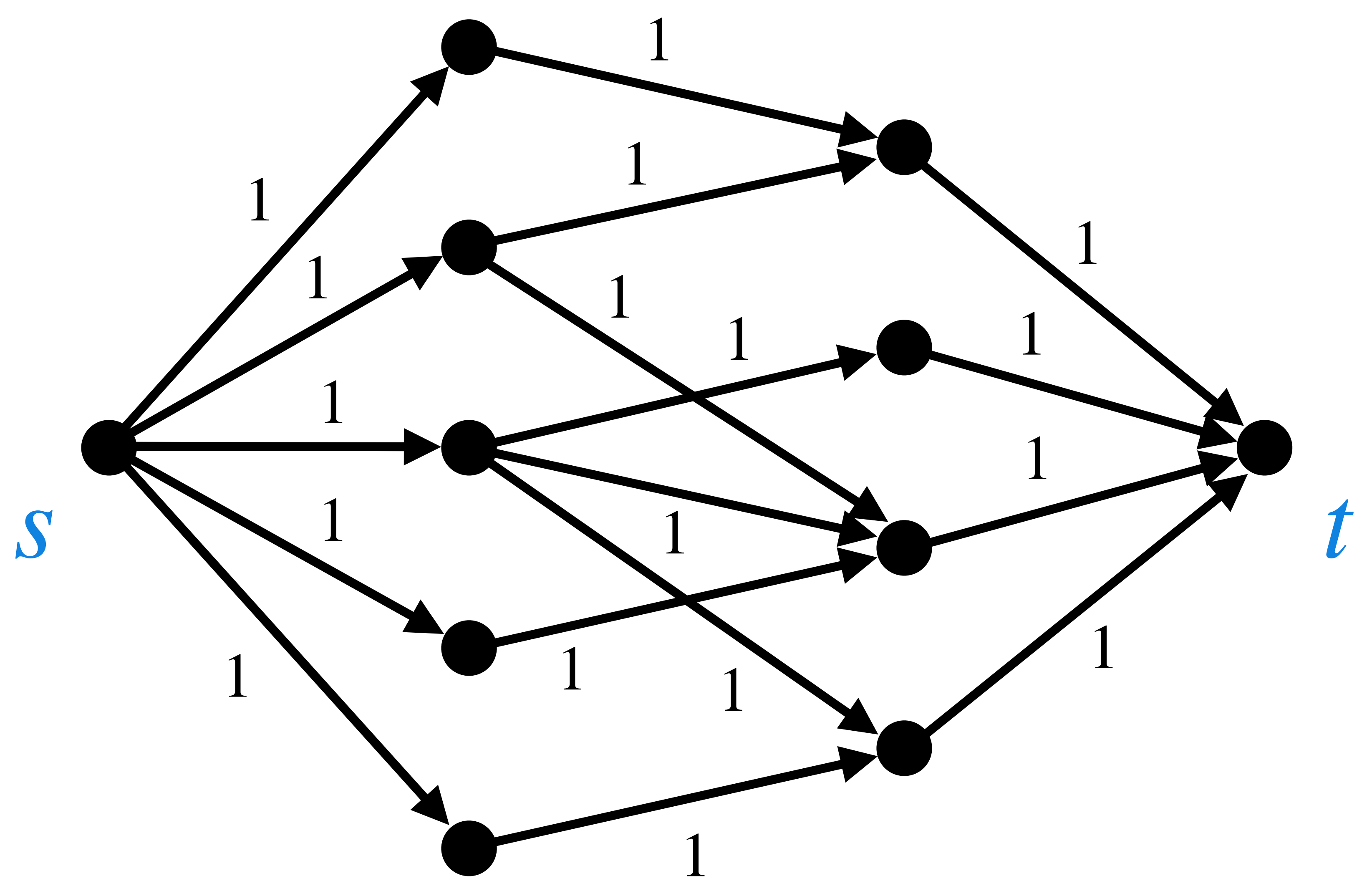

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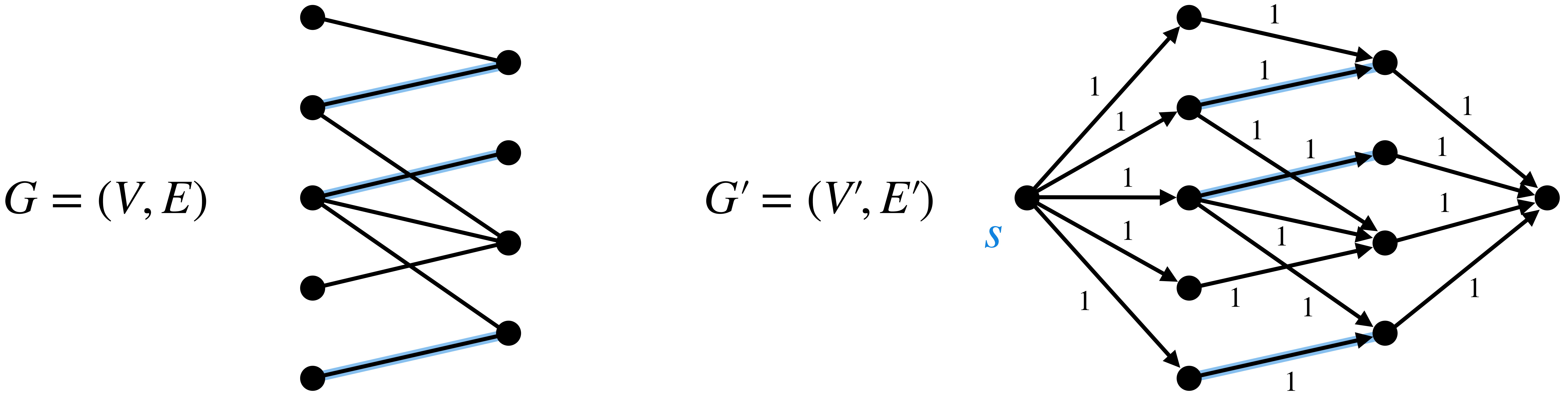

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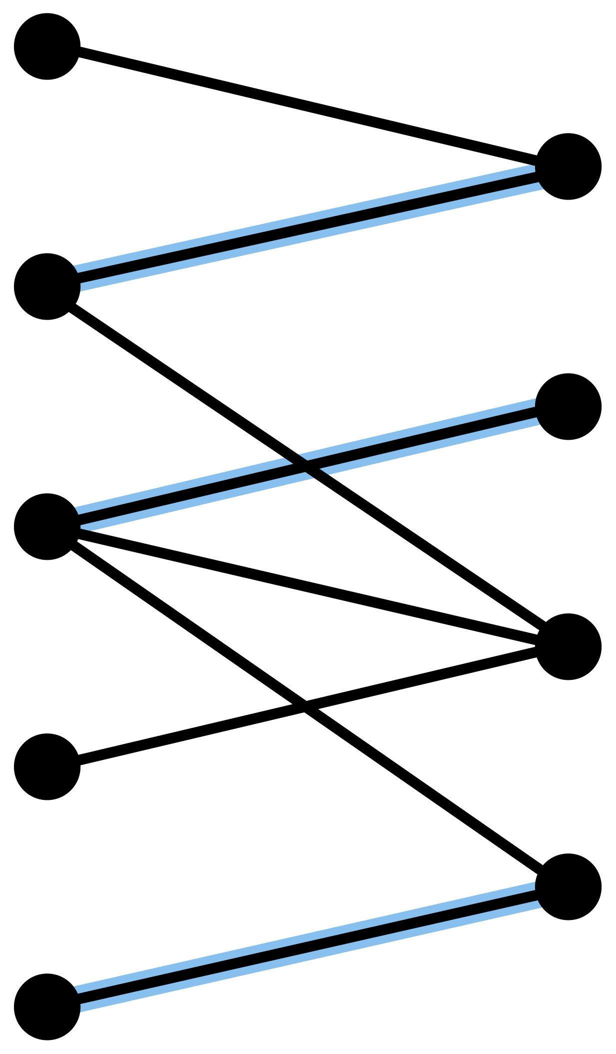

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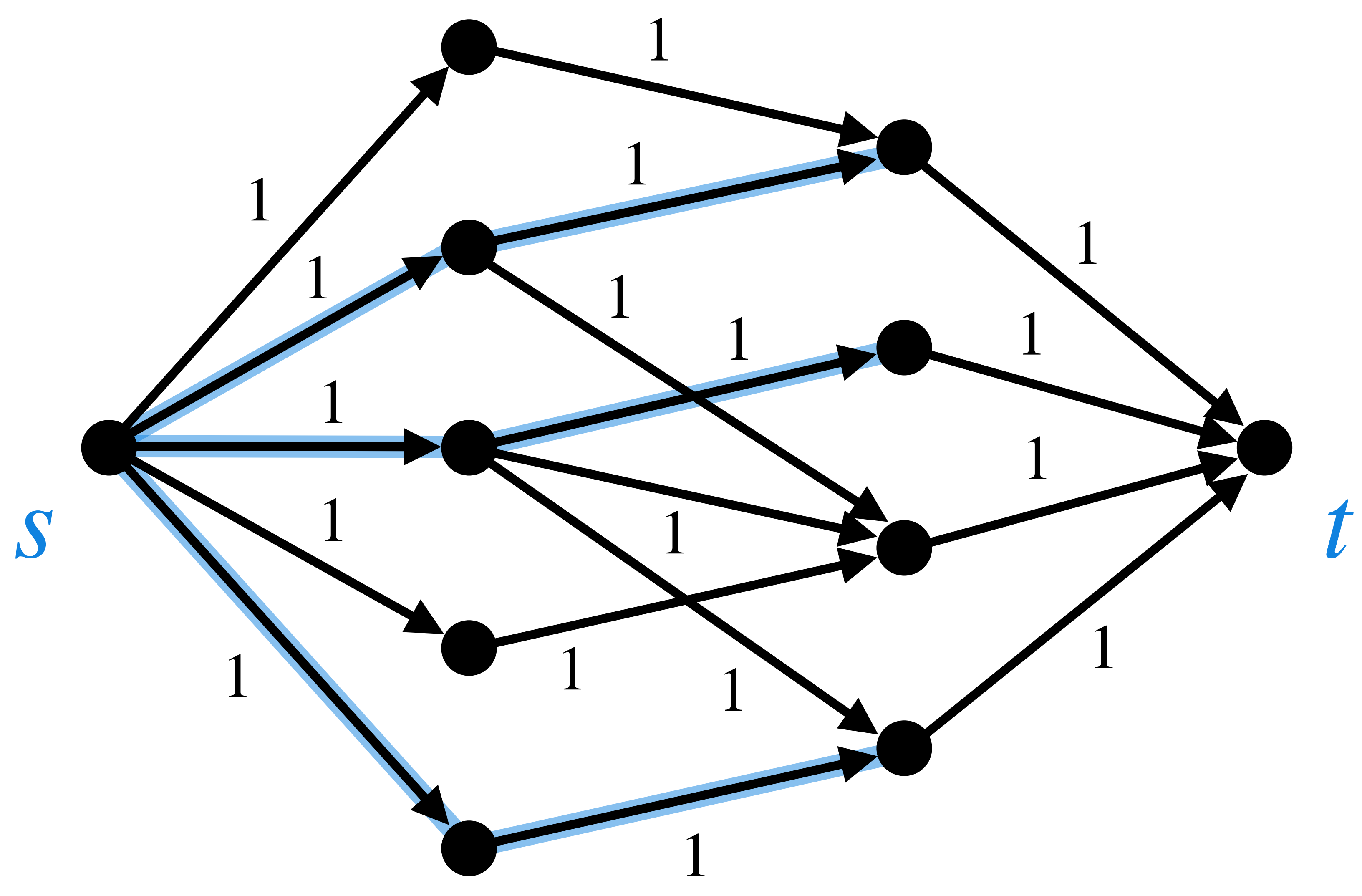

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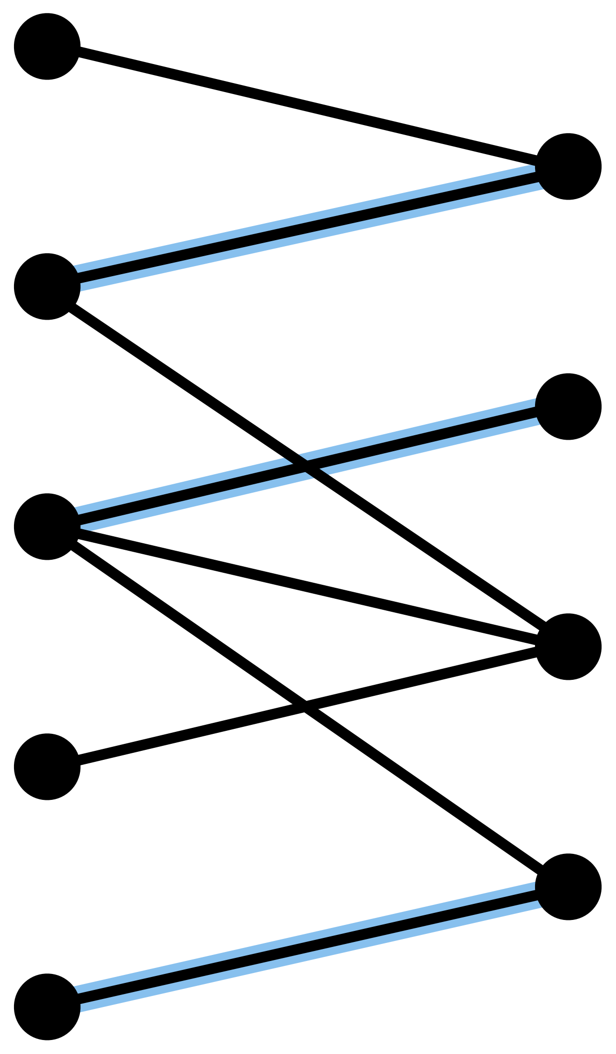

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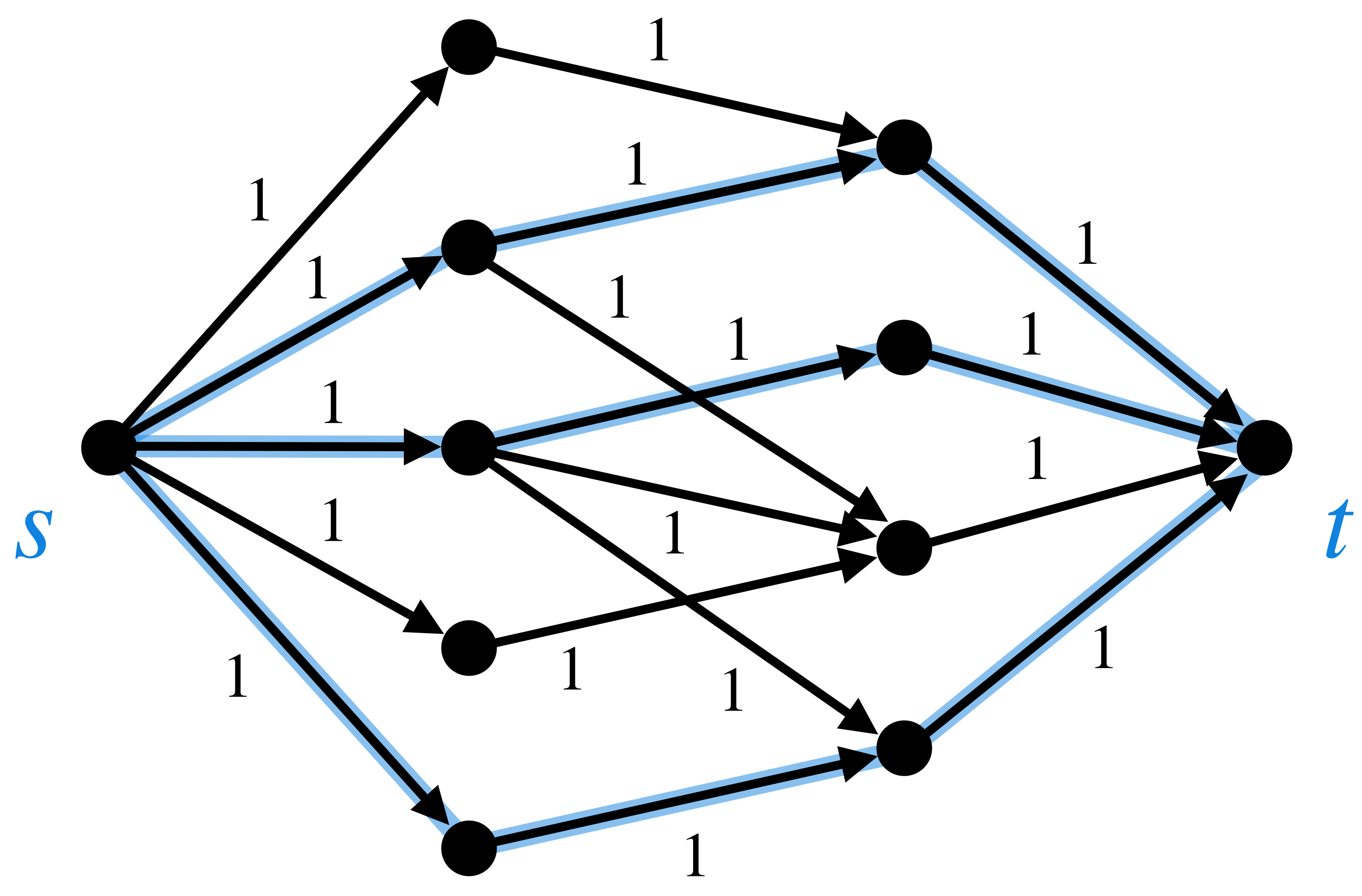

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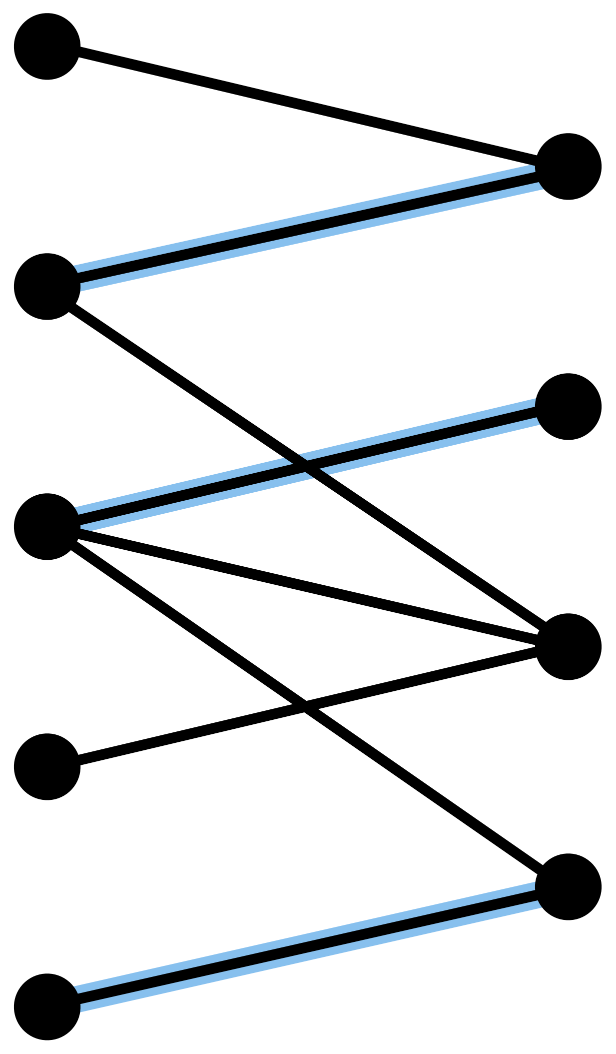

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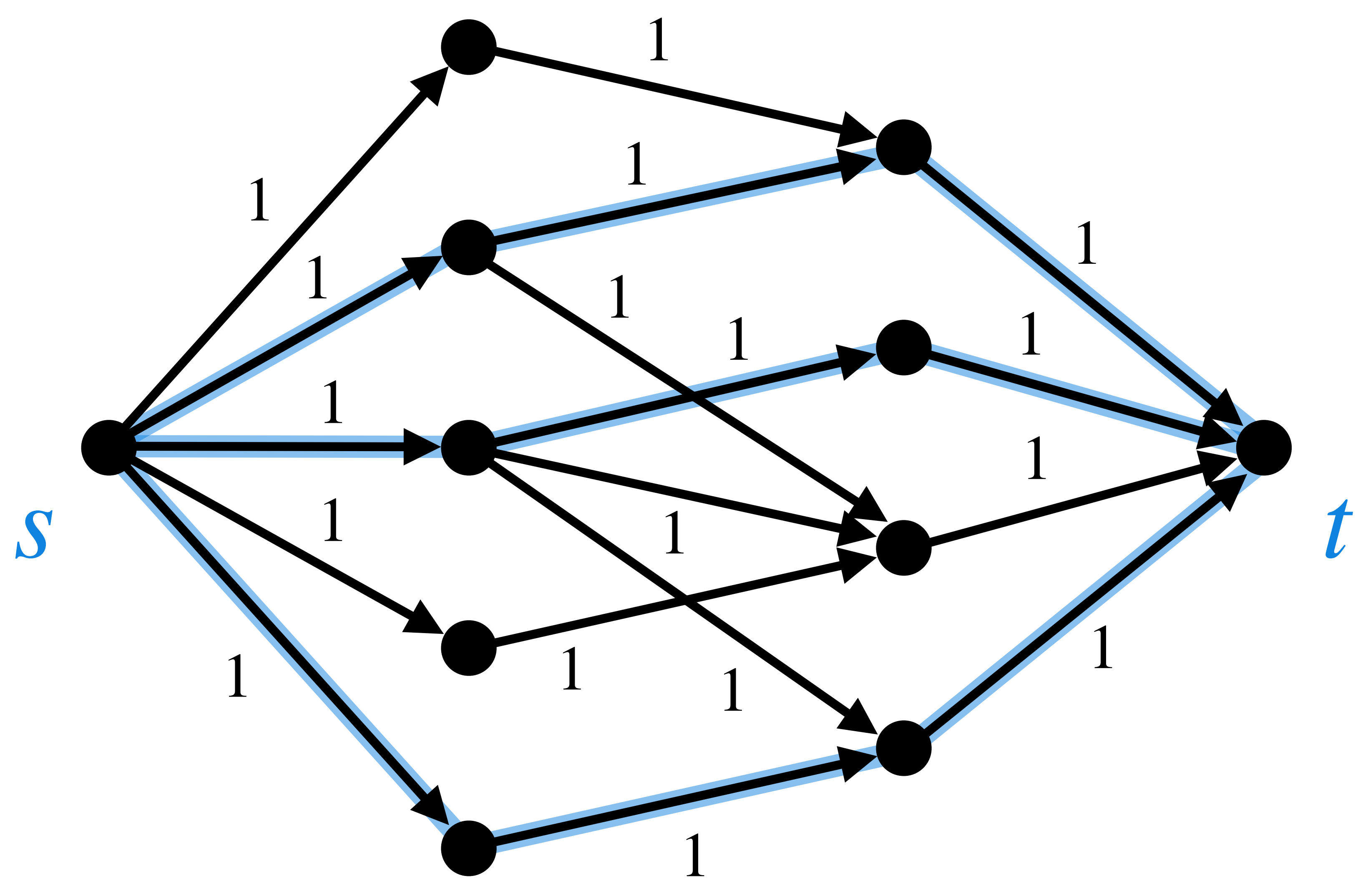

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Claim: If M is a matching in G, then there is an **integer-valued** f in G' with value $|f| = |M|$.

Proof: Consider the following flow f such that $|f| = |M|$:

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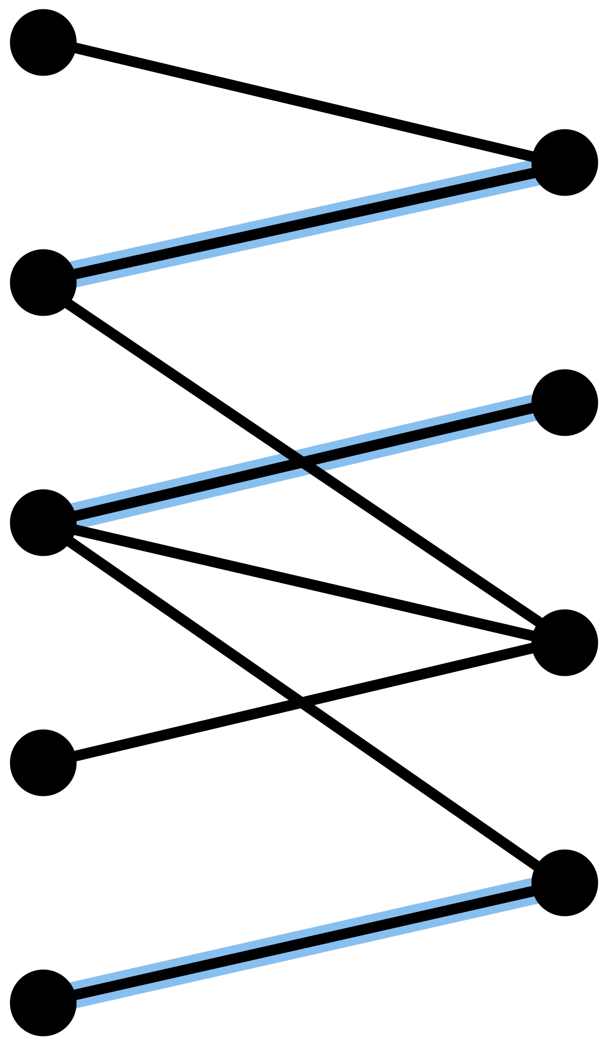

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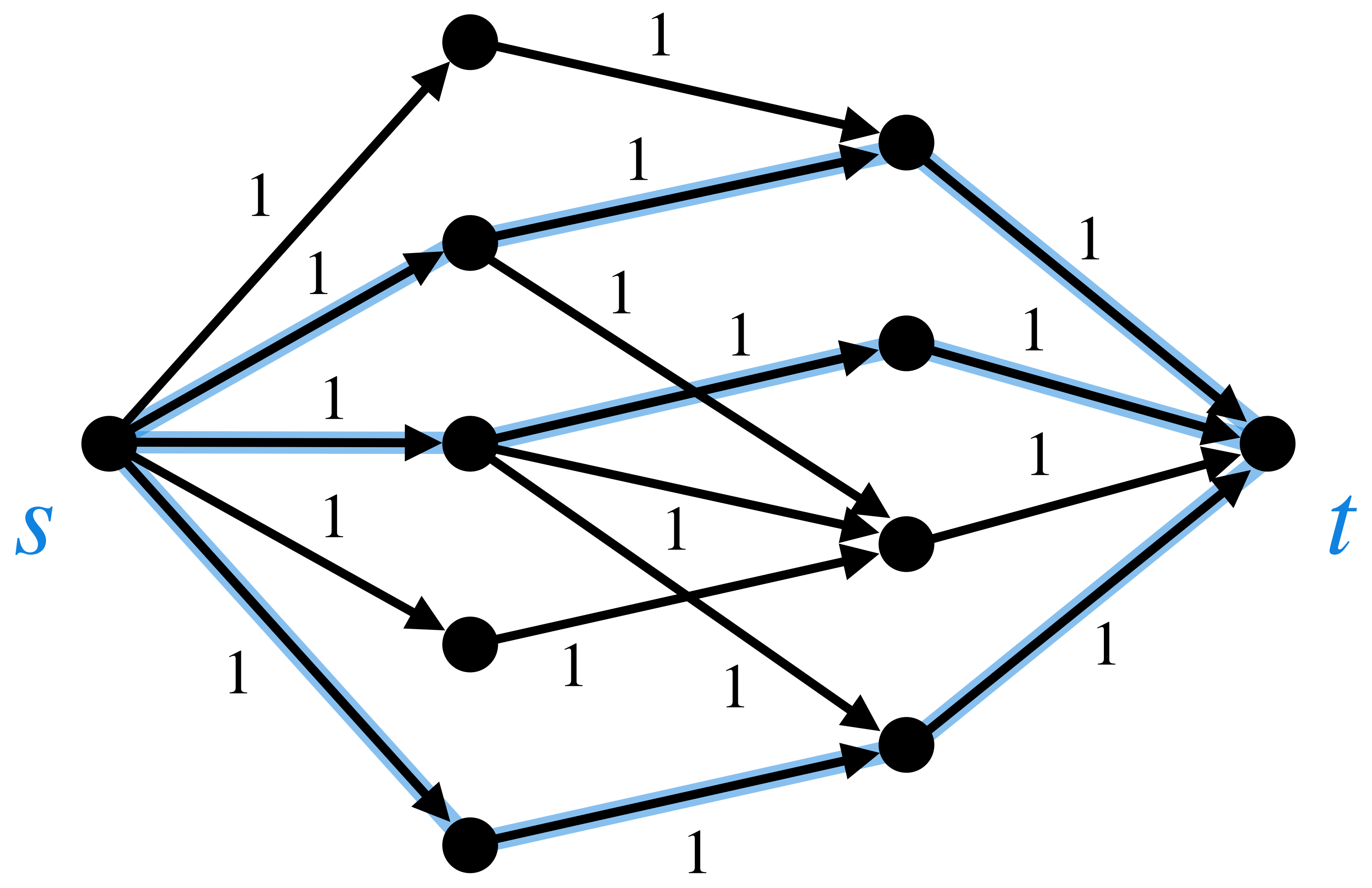

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Proof: Consider the following flow f such that $|f| = |M|$:

- If $(u, v) \in M$, then $f(u, v) = f(s, u) = f(v, t) = 1$

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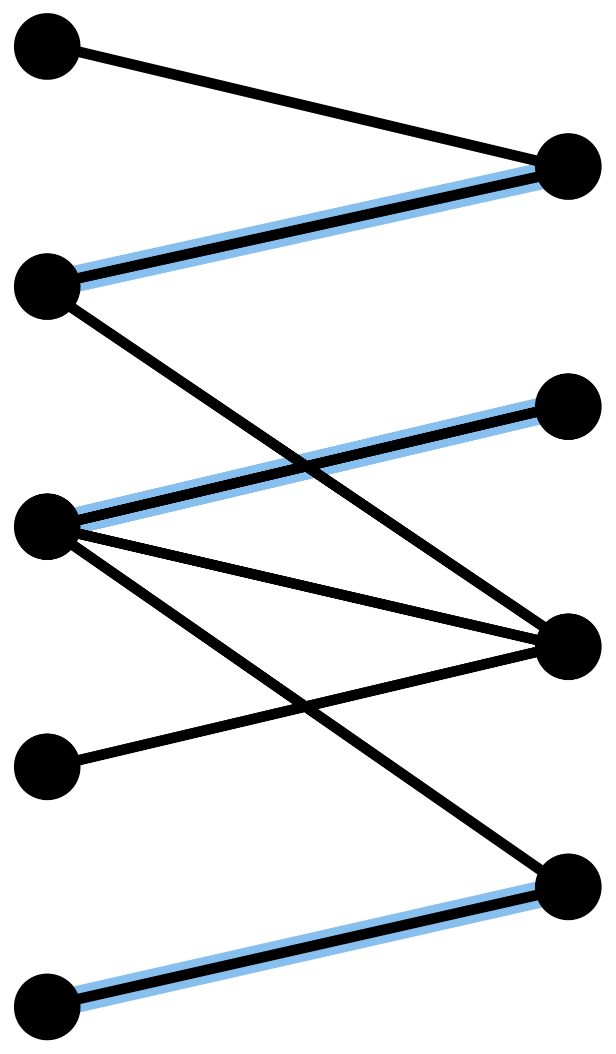

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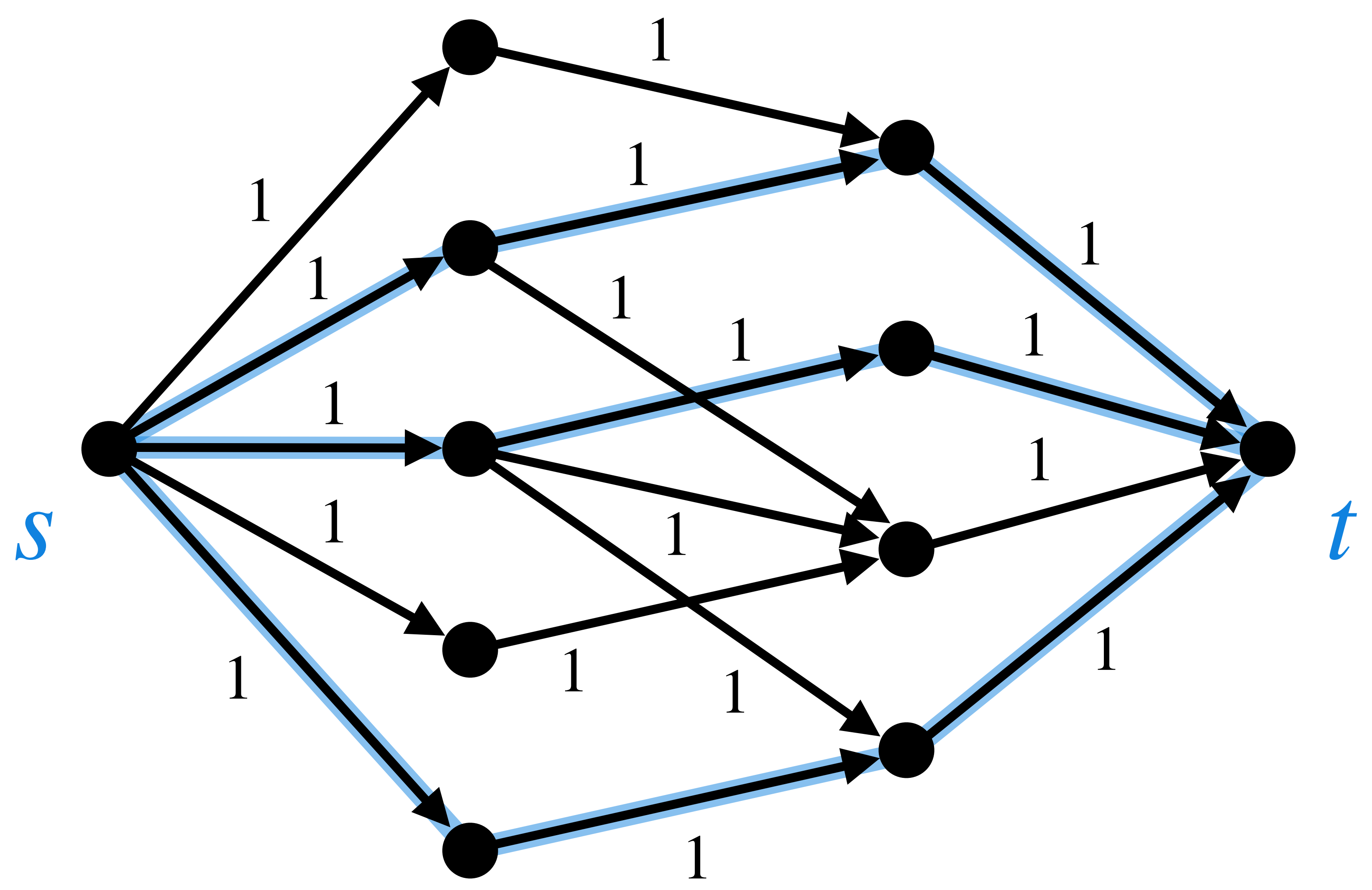

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- If $(u, v) \in M$, then $f(u, v) = f(s, u) = f(v, t) = 1$
- For other (u, v) edges in E', $f(u, v) = 0$.

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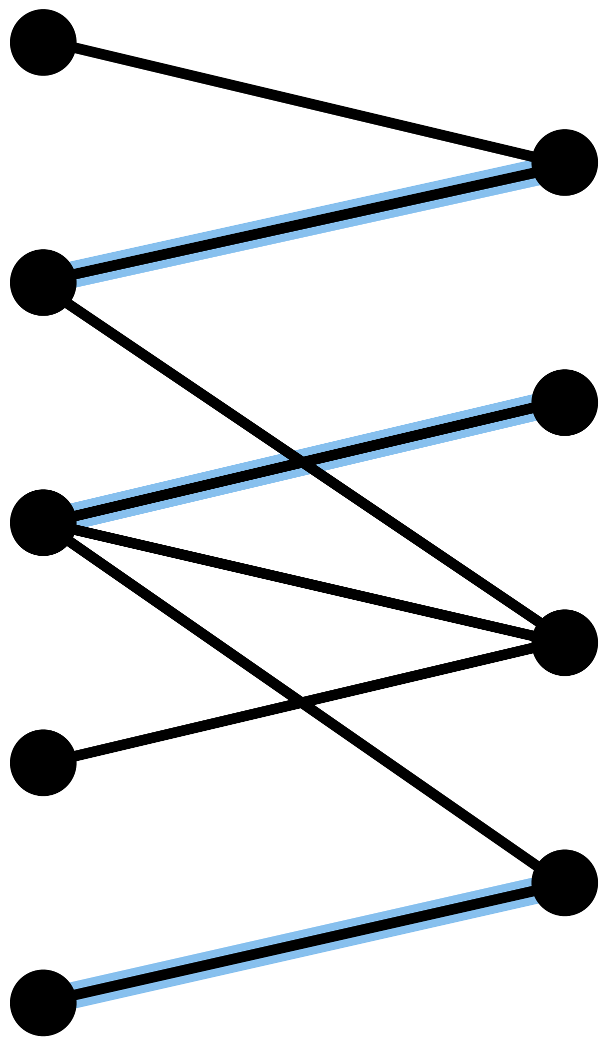

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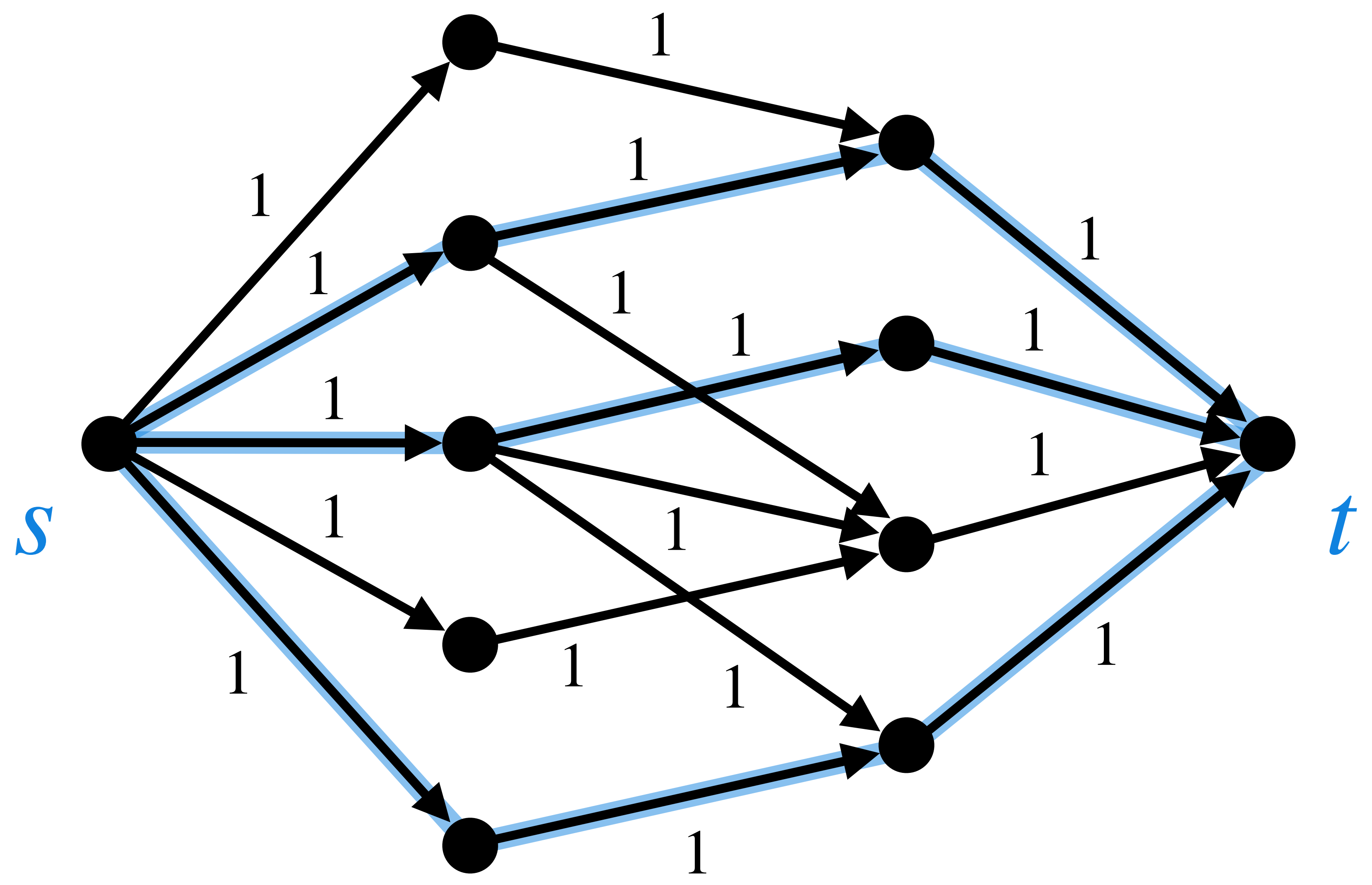

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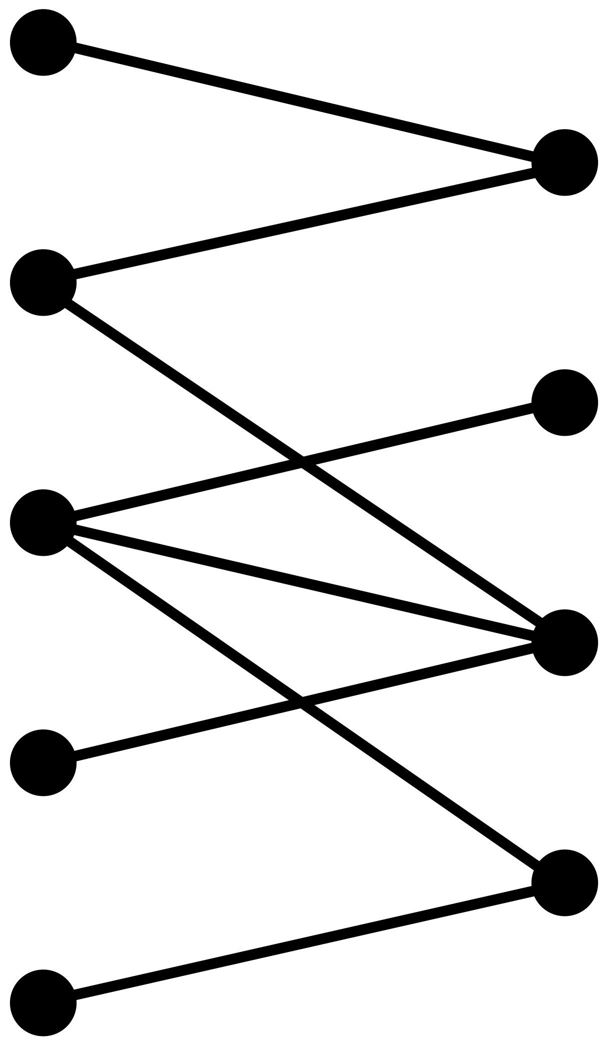

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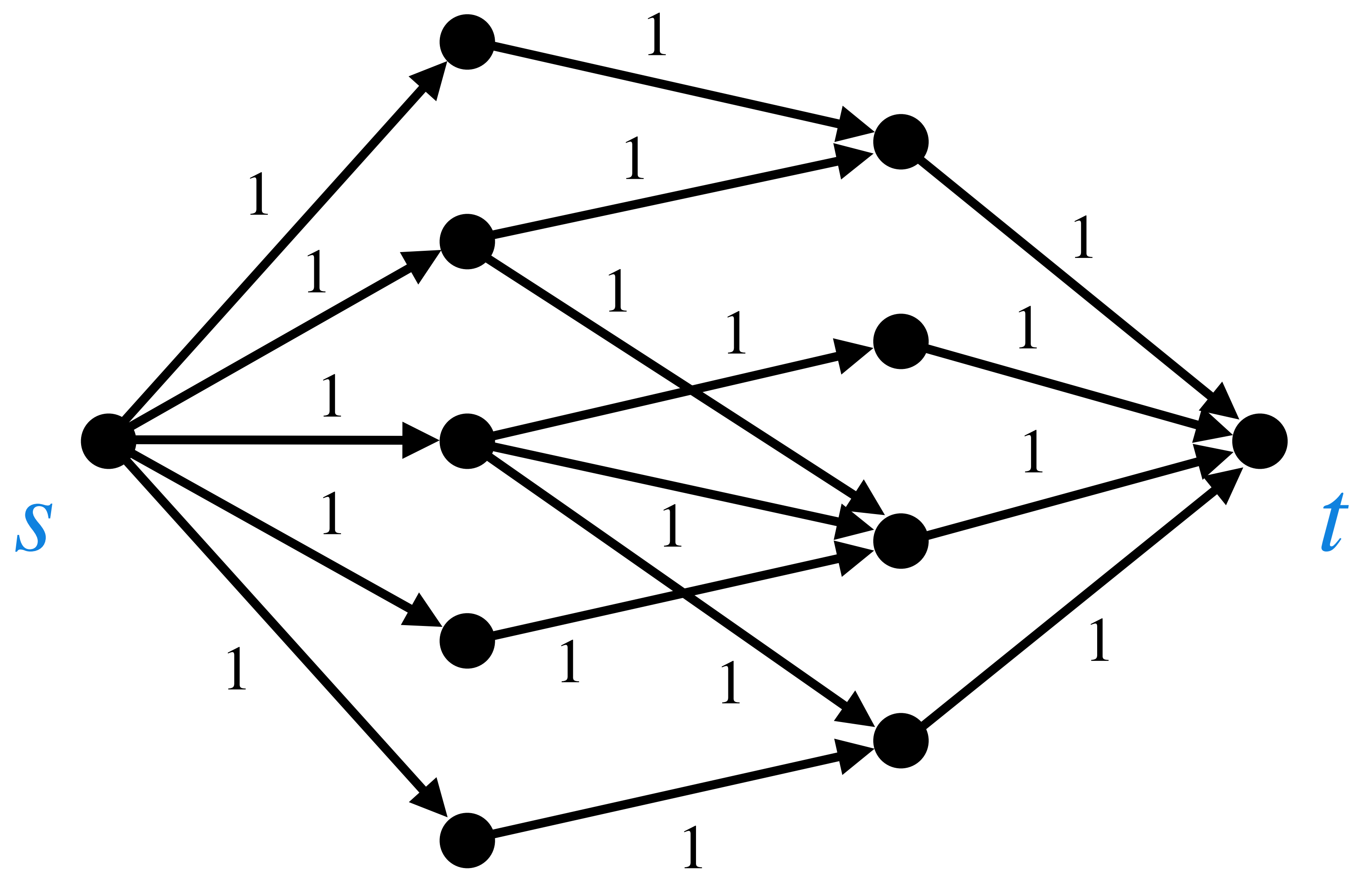

Bipartite Matching to Flow

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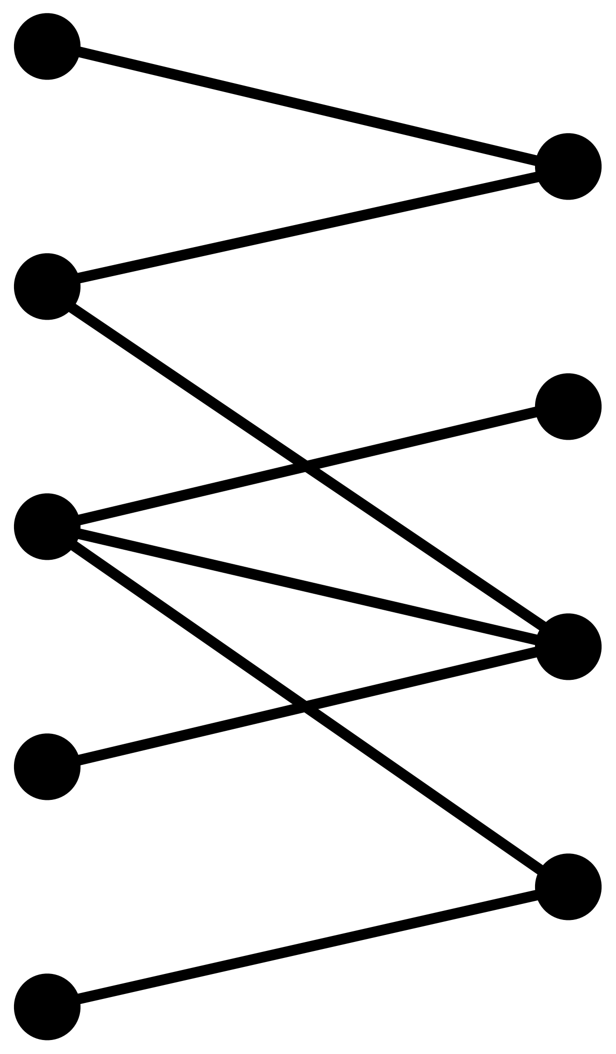

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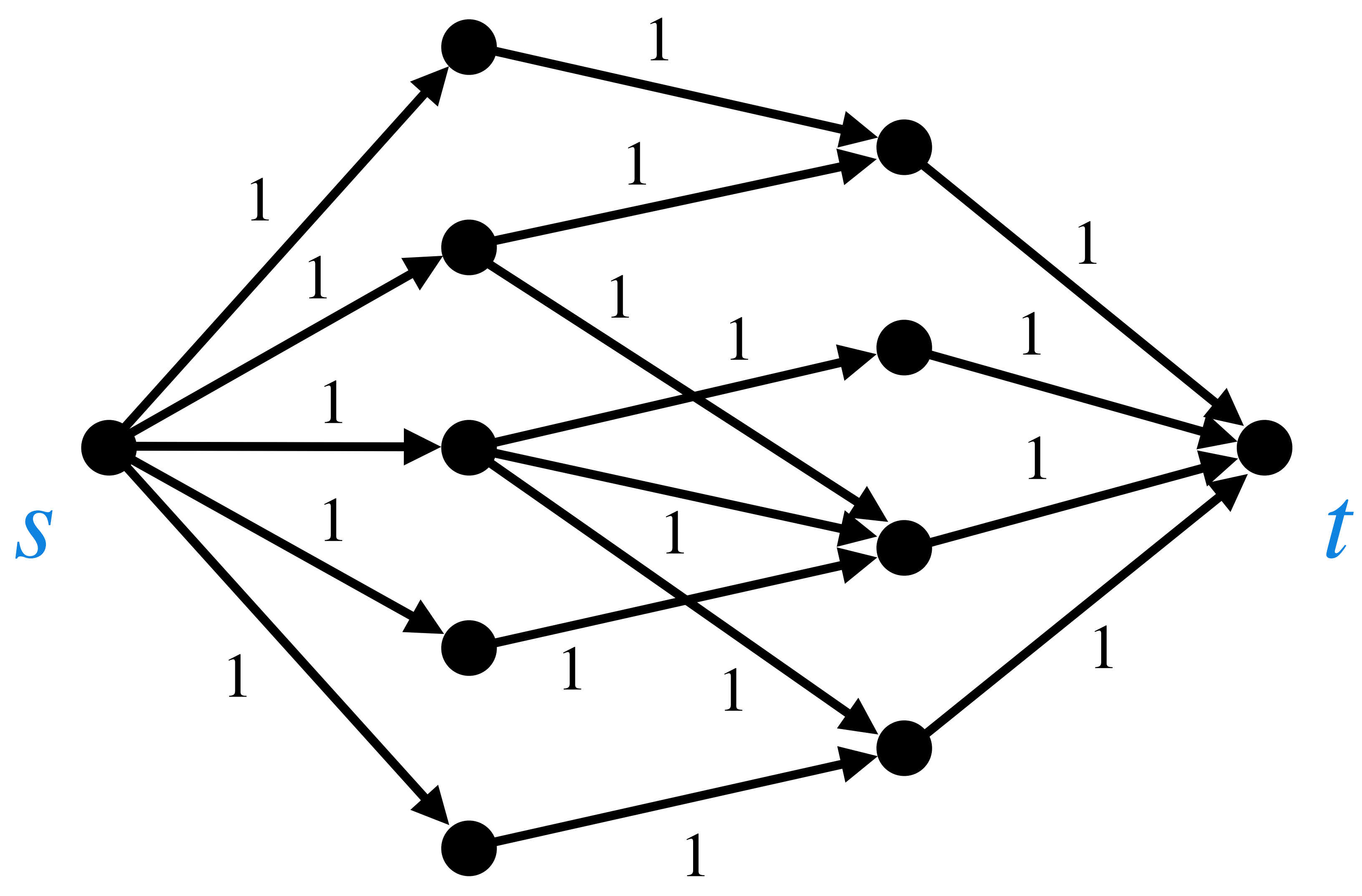

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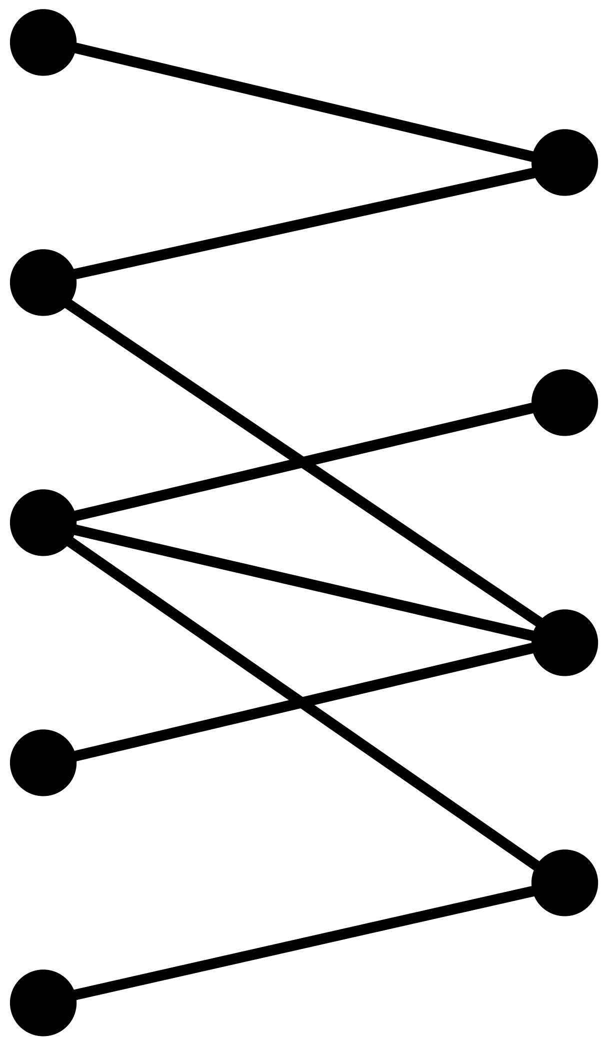

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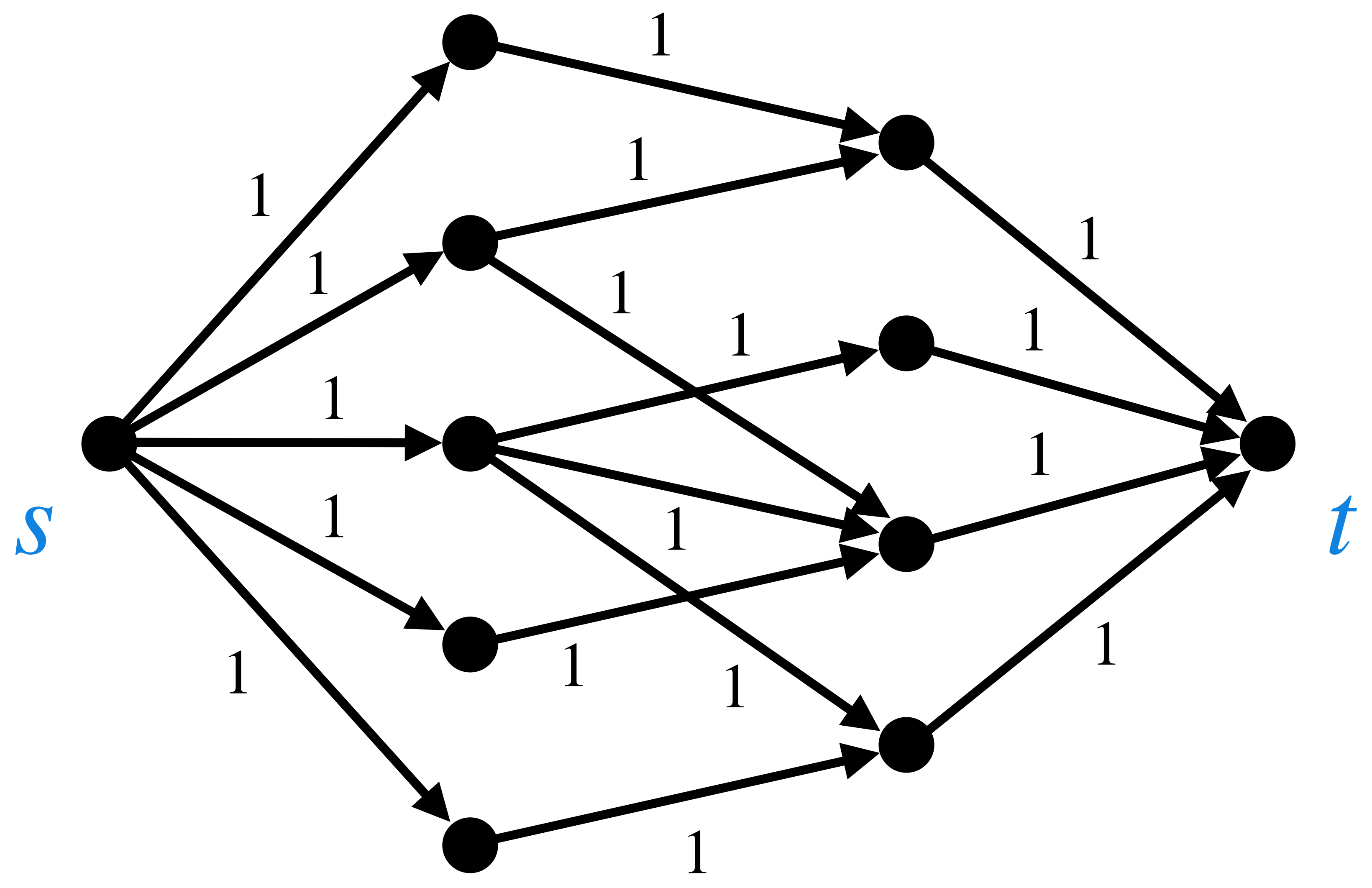

Bipartite Matching to Flow

Claim: If f is an **integer-valued** flow in G', then $\exists$ a matching M of size $|f|$ in G.

Proof:

Can two edges with flow one
from L to R have a common vertex?

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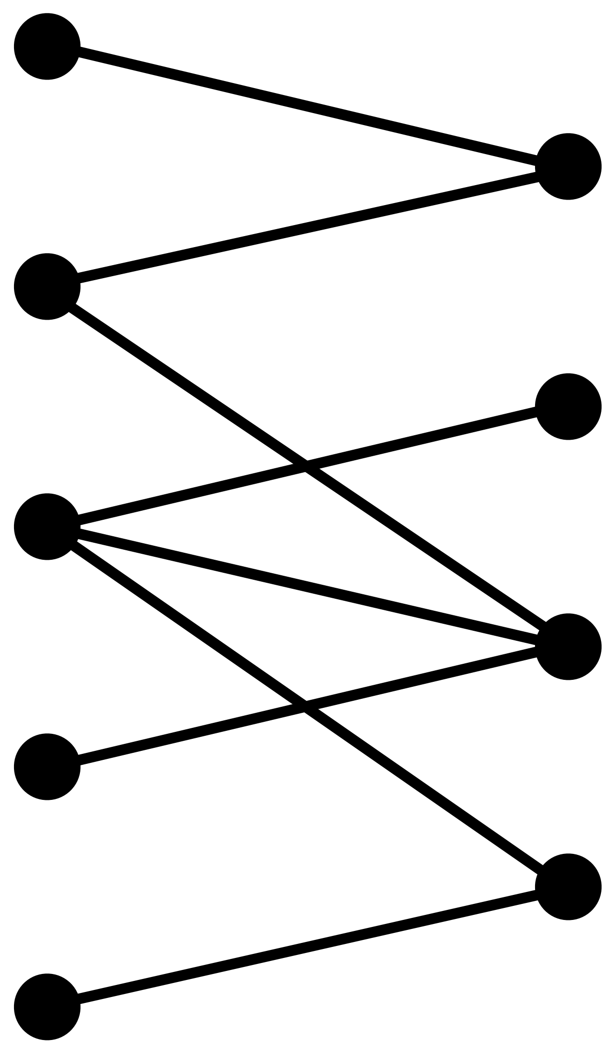

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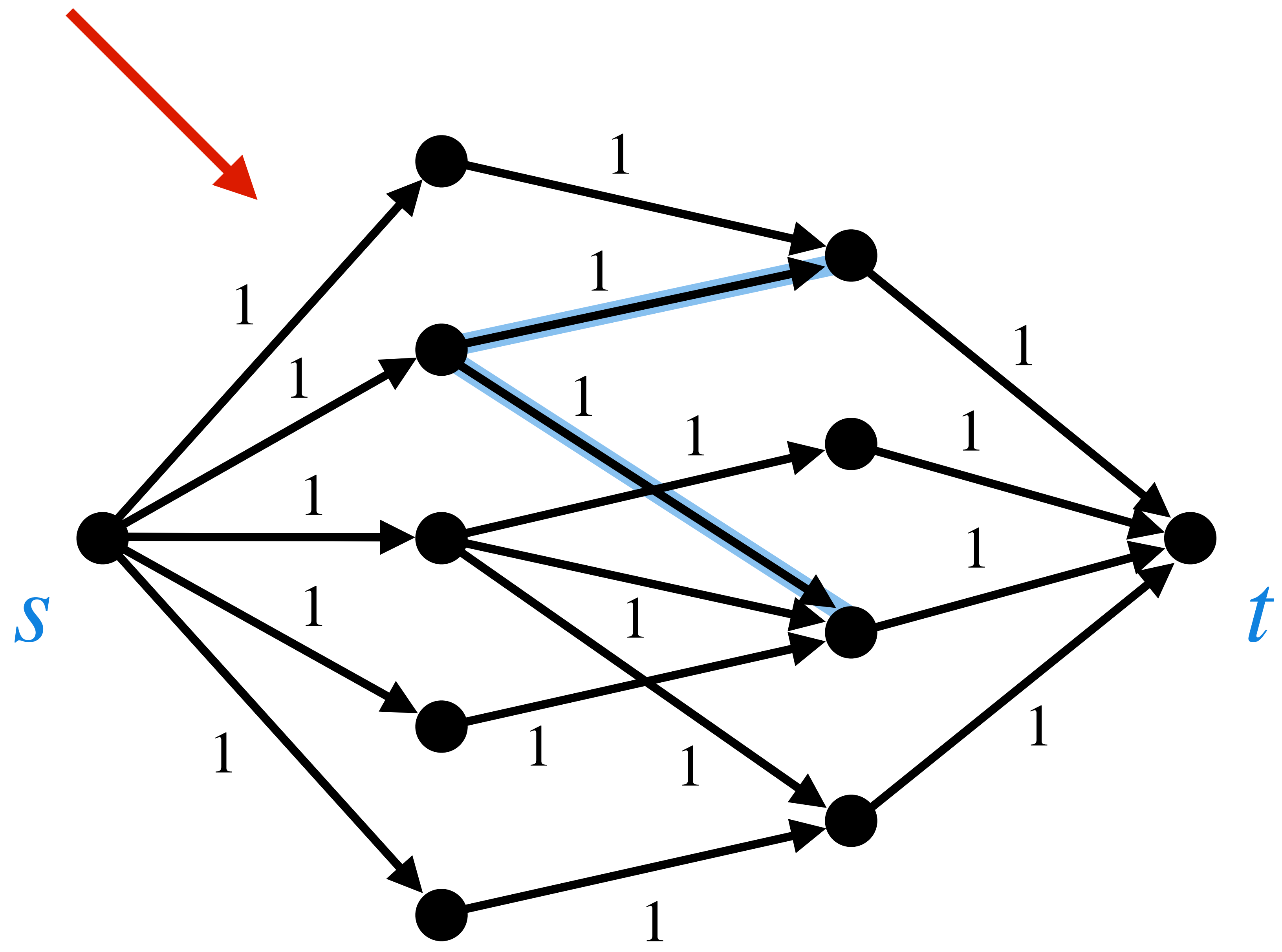

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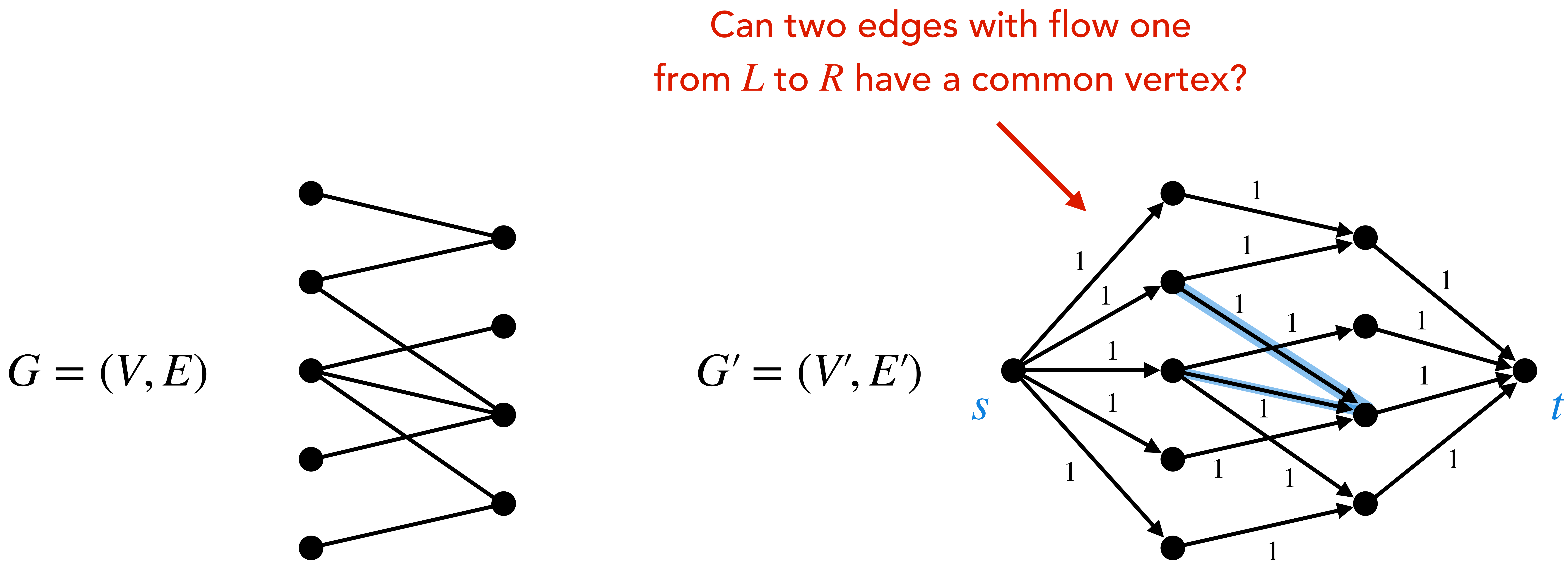

Bipartite Matching to Flow

Claim: If f is an **integer-valued** flow in G', then ∃ a matching M of size $|f|$ in G .

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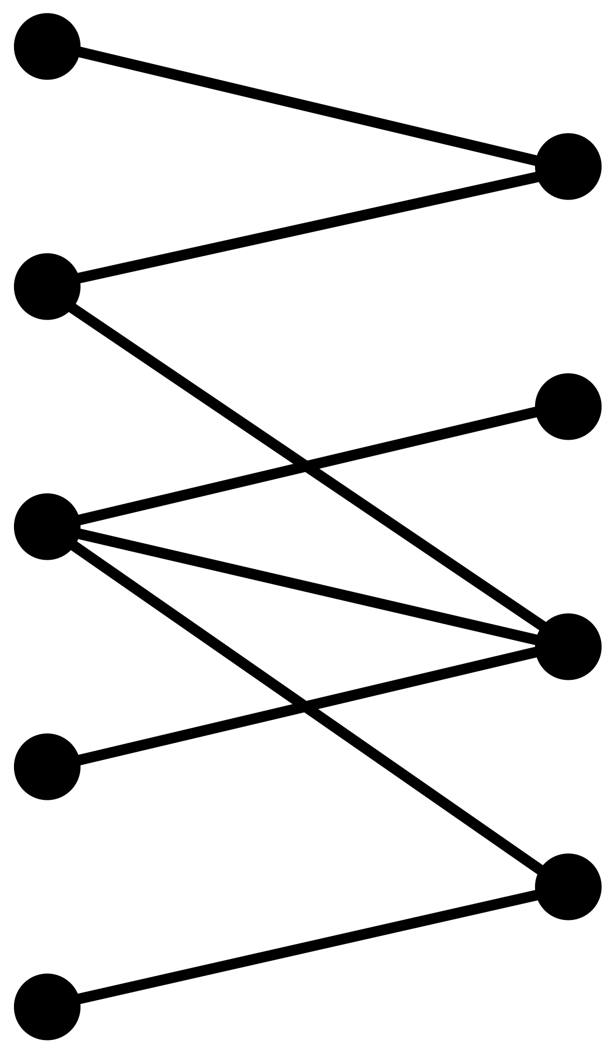

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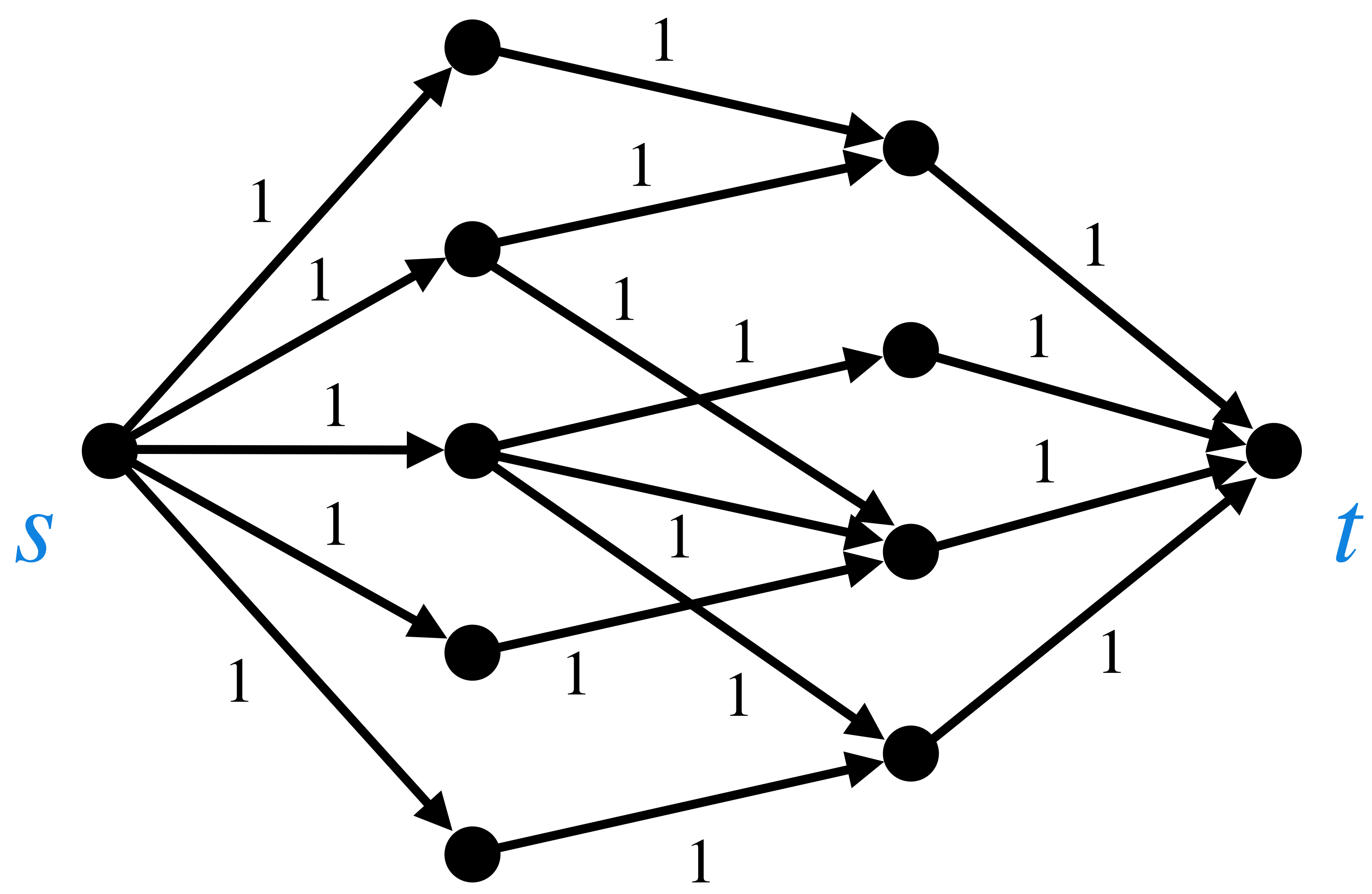

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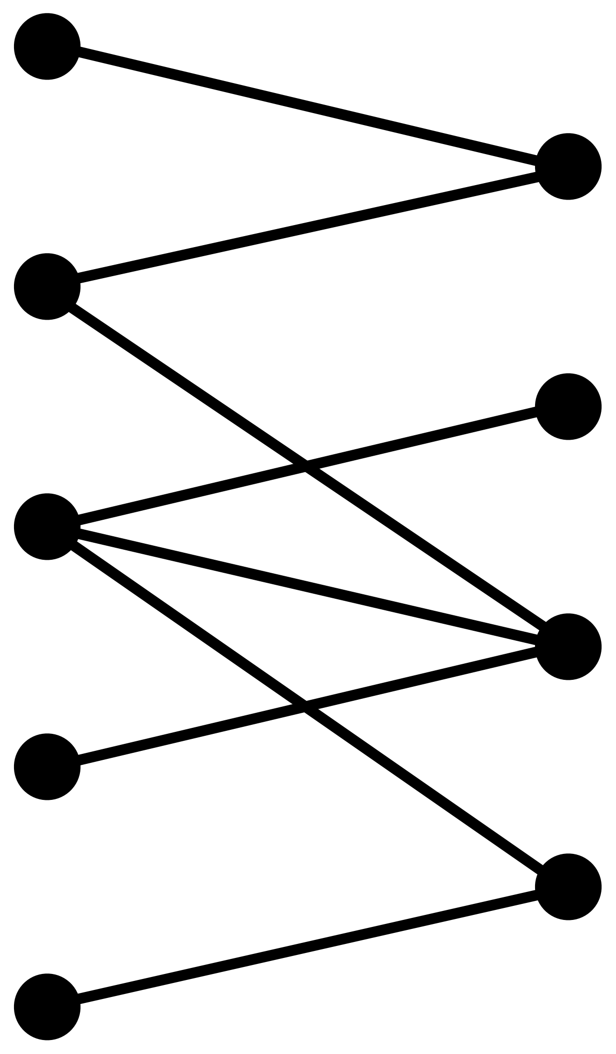

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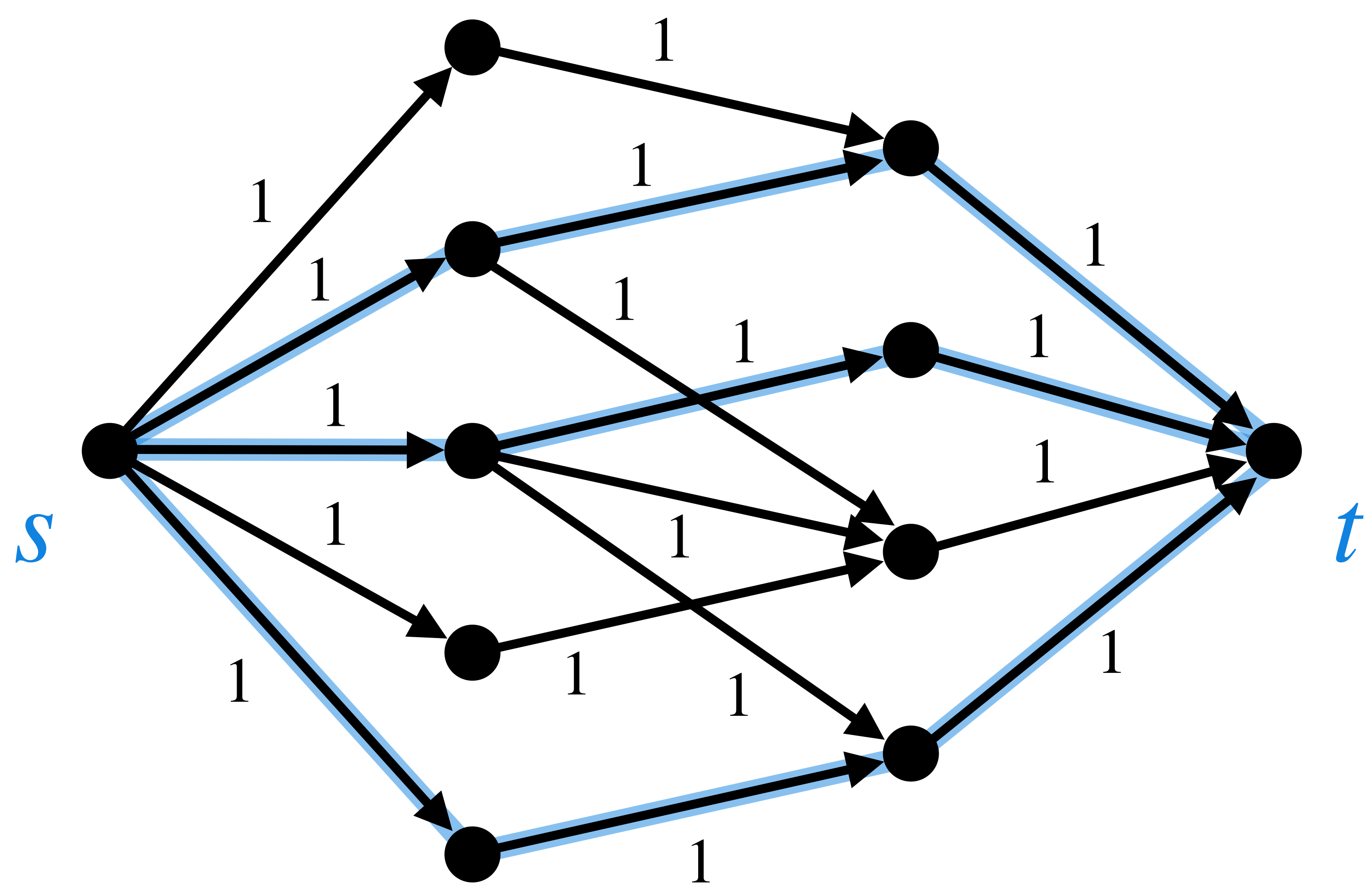

Bipartite Matching to Flow

Claim: If f is an **integer-valued** flow in G', then $\exists$ a matching M of size $|f|$ in G.

Proof:

Pick corresponding edges to those who have flow 1 from L to R.

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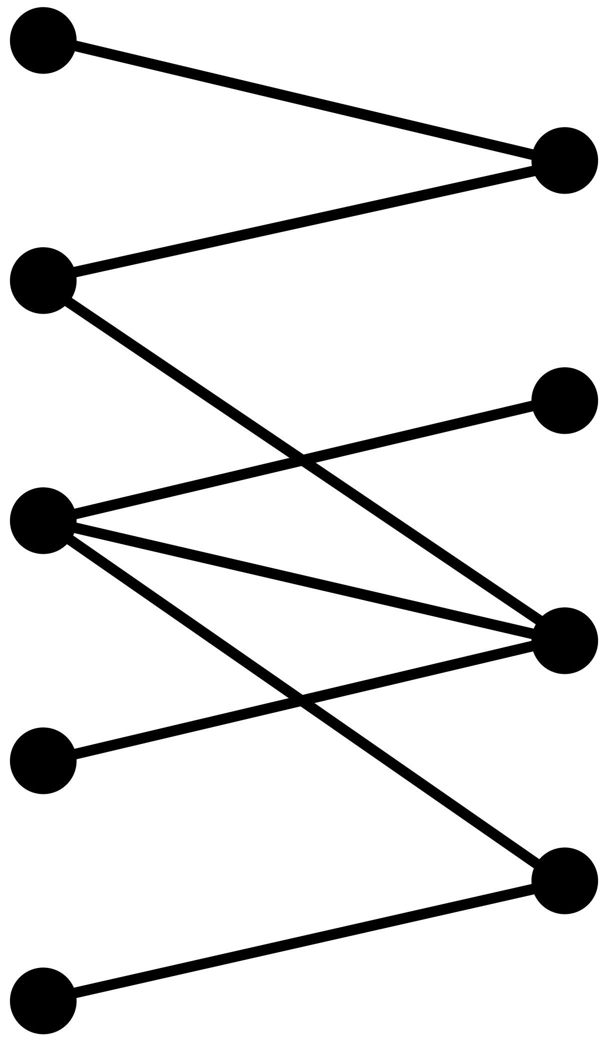

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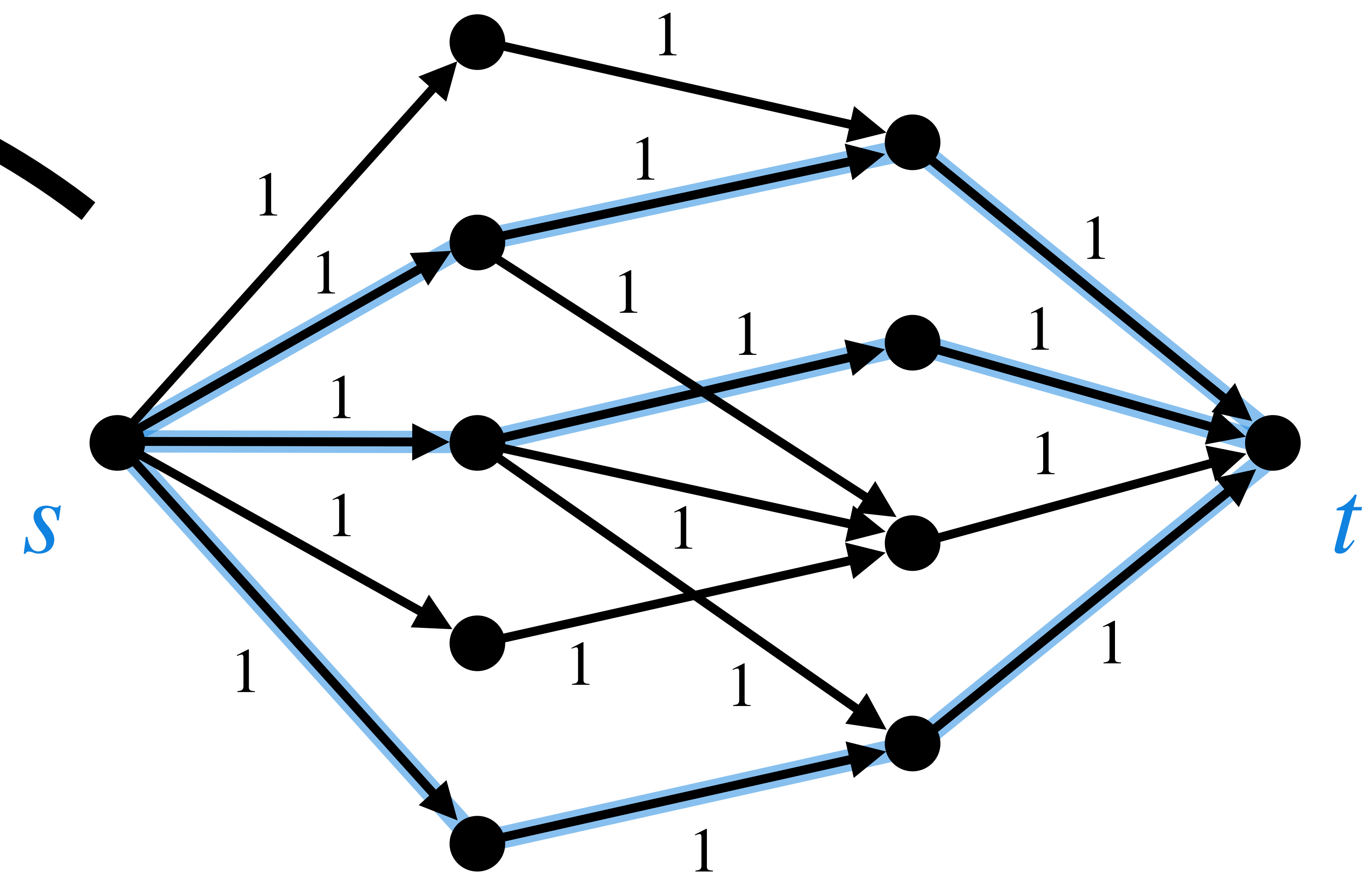

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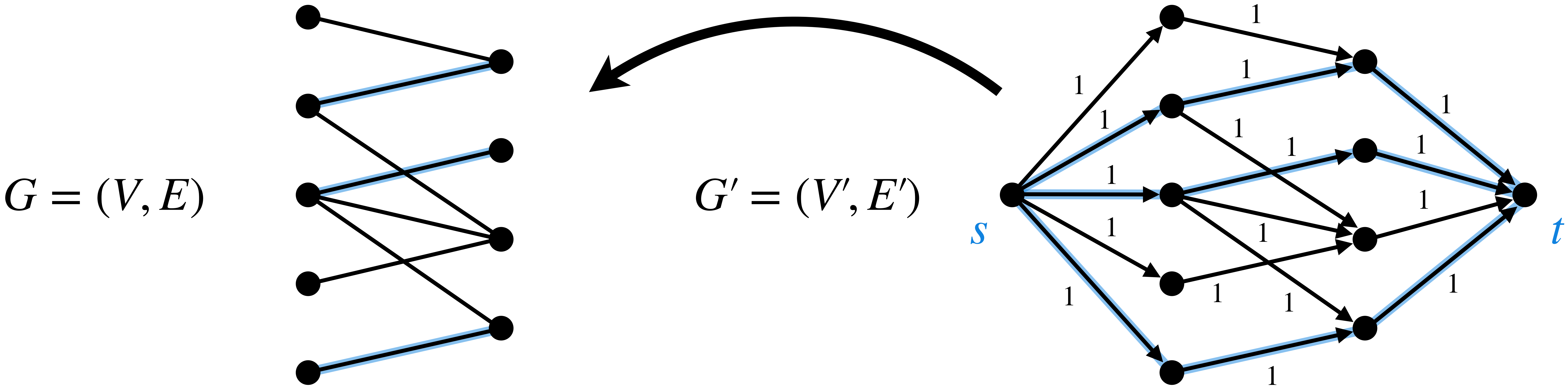

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$|M|$

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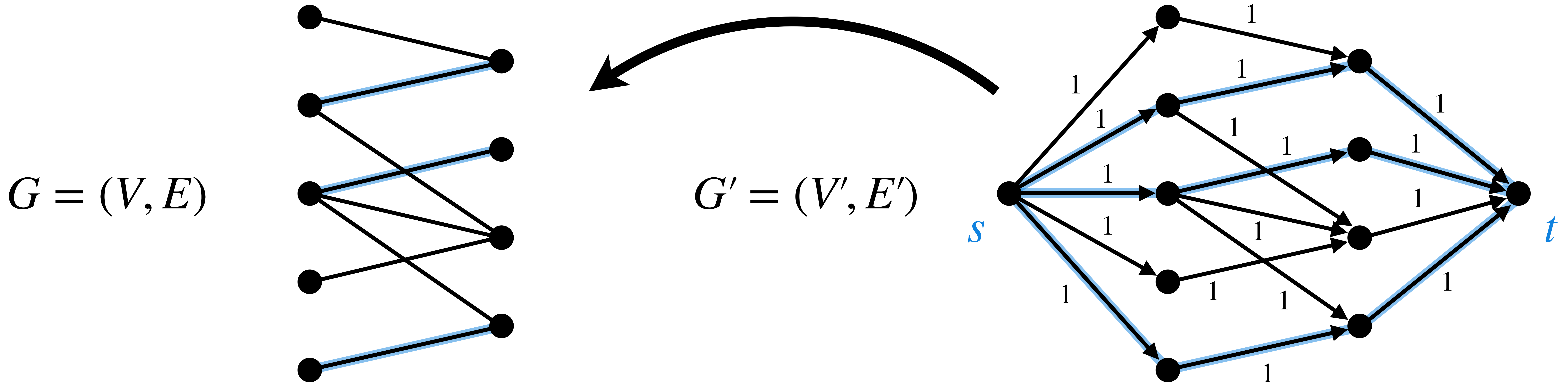

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$|M| =$ # of edges from L to R with flow 1

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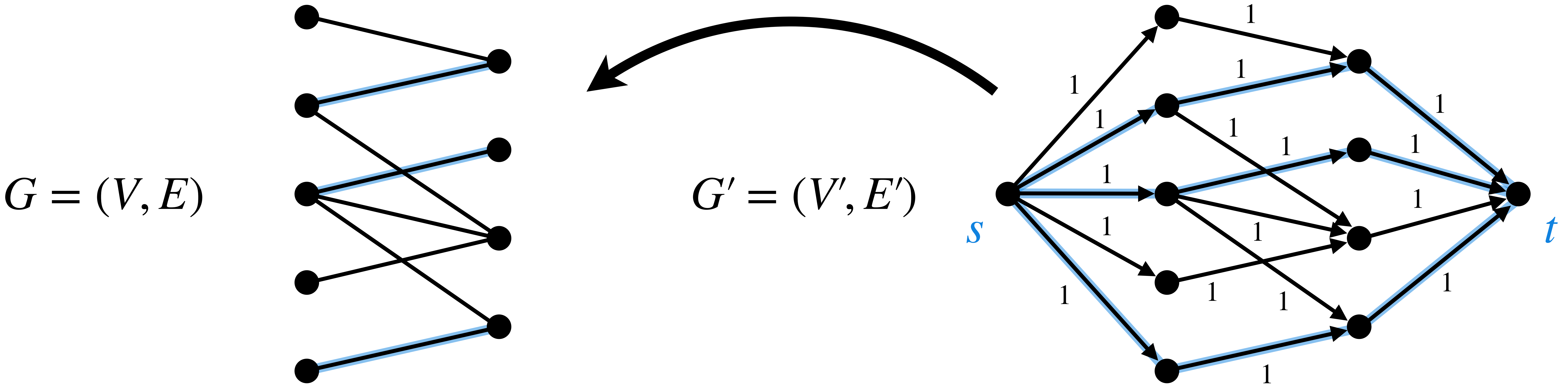

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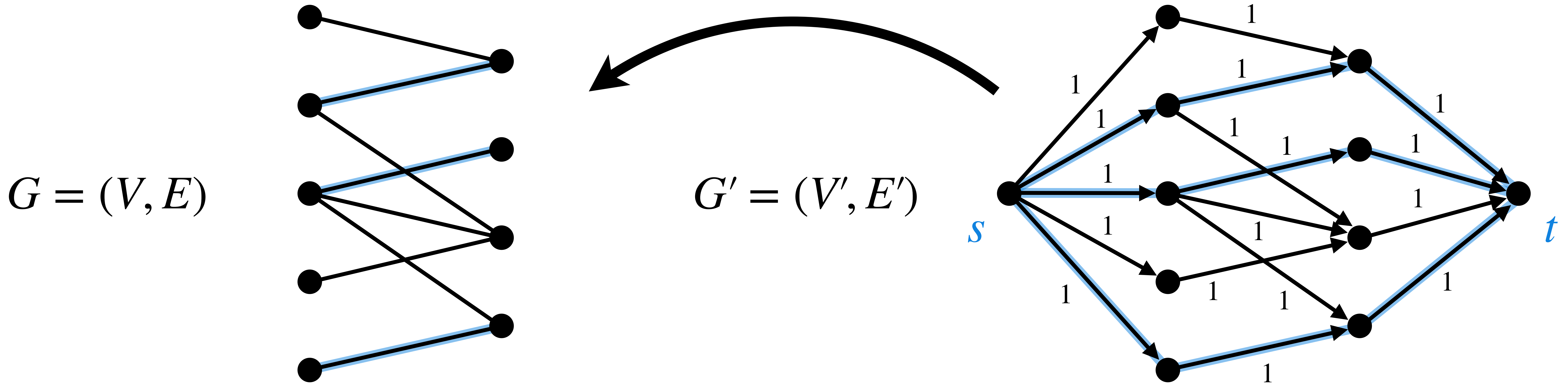

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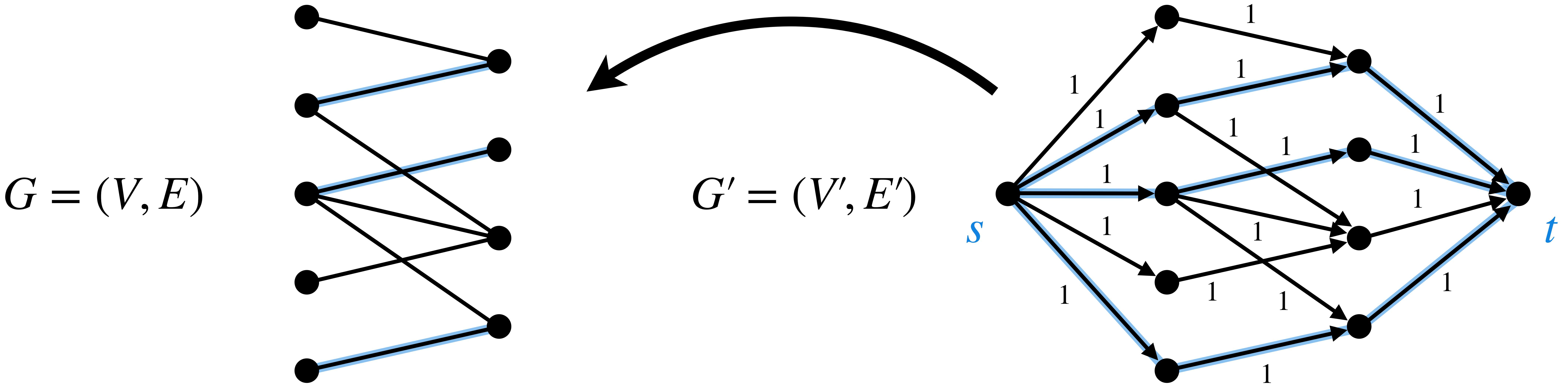

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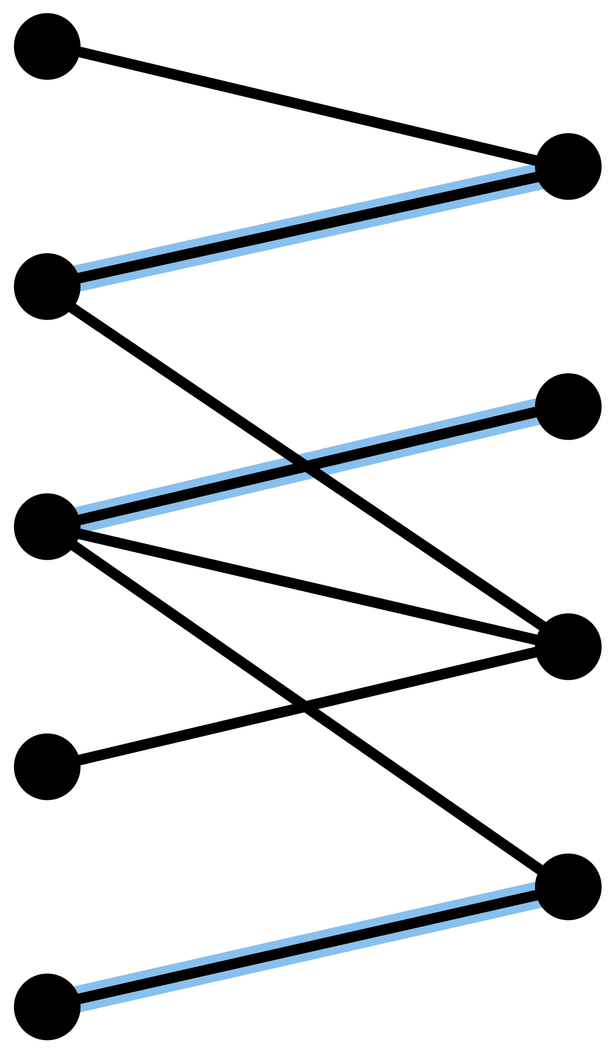

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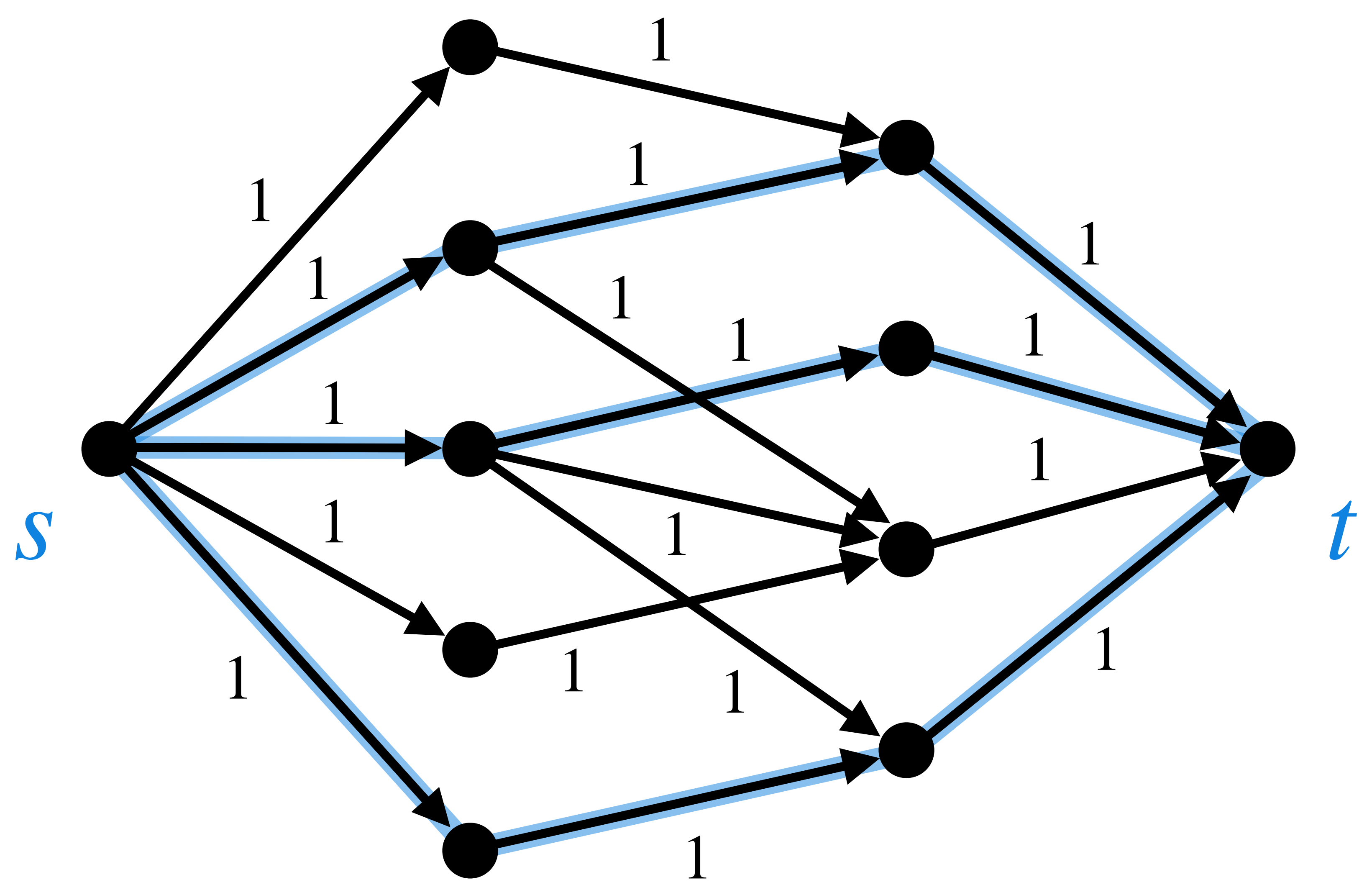

Bipartite Matching to Flow

Claim: If f is an **integer-valued** flow in G', then $\exists$ a matching M of size $|f|$ in G .

Proof: M in G contains edges from L to R which have flow 1 in G'.

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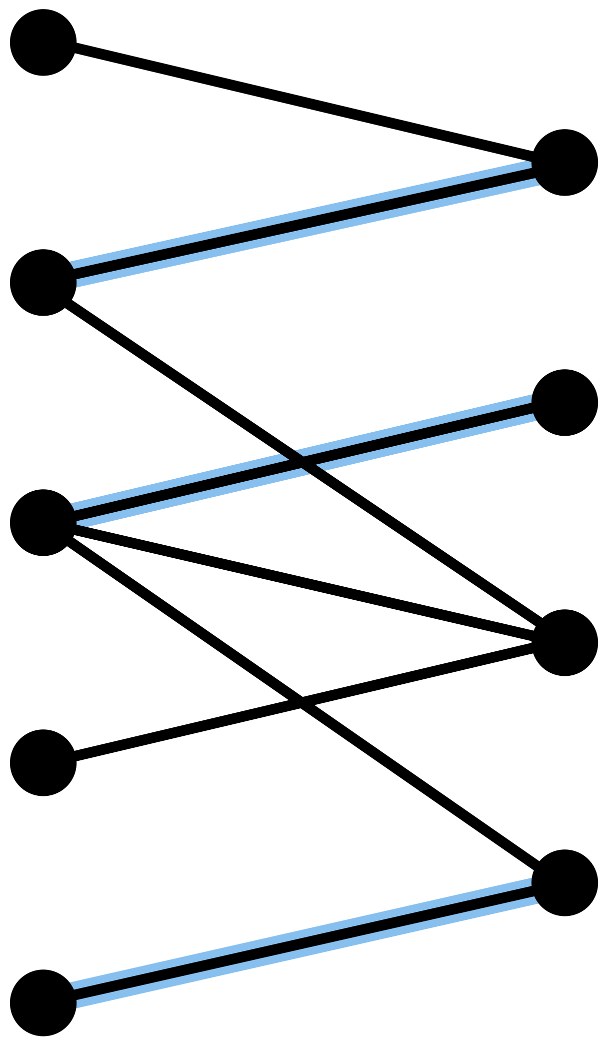

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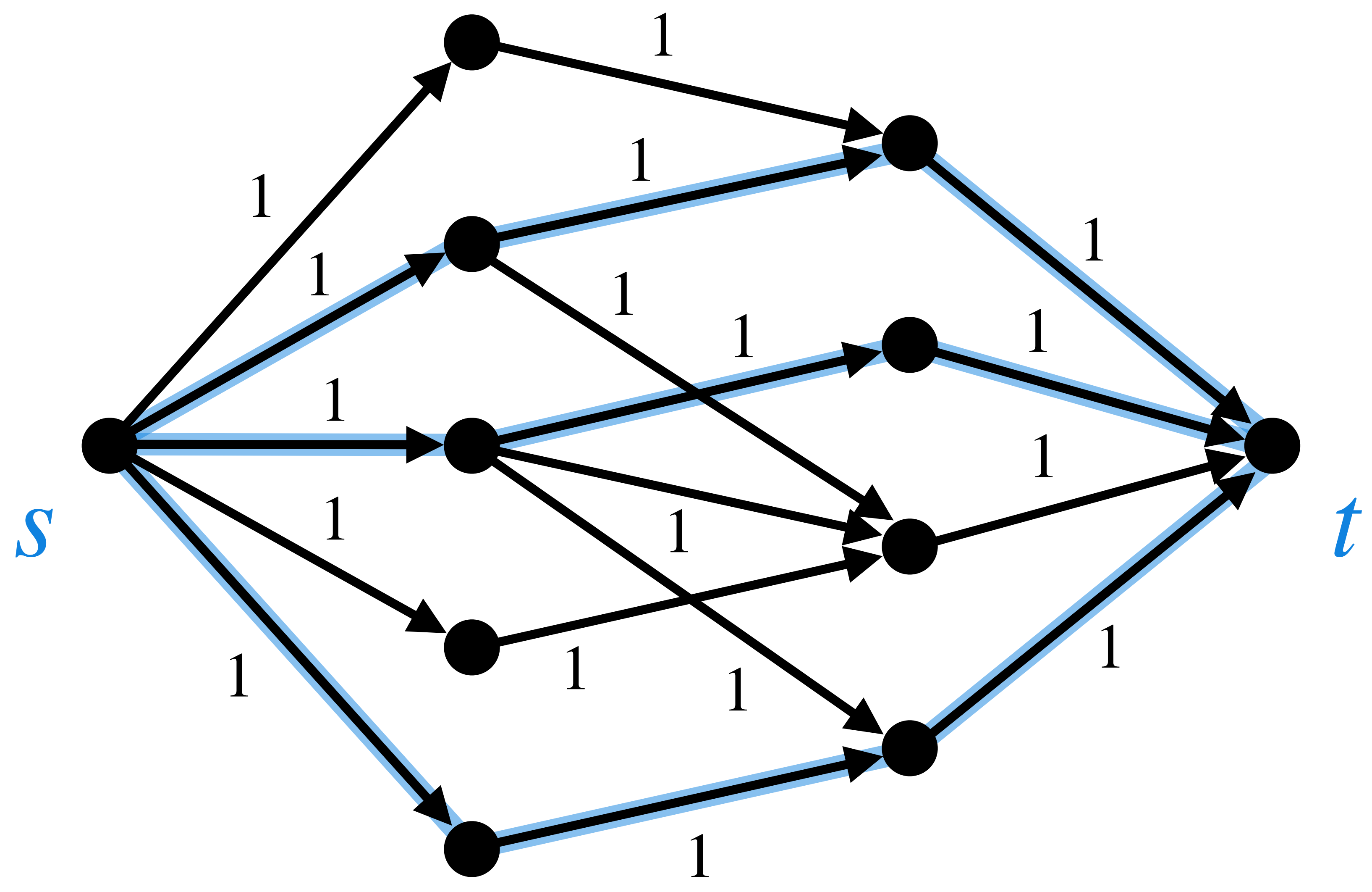

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Prove it yourself why M will be a matching of size $|f|$.

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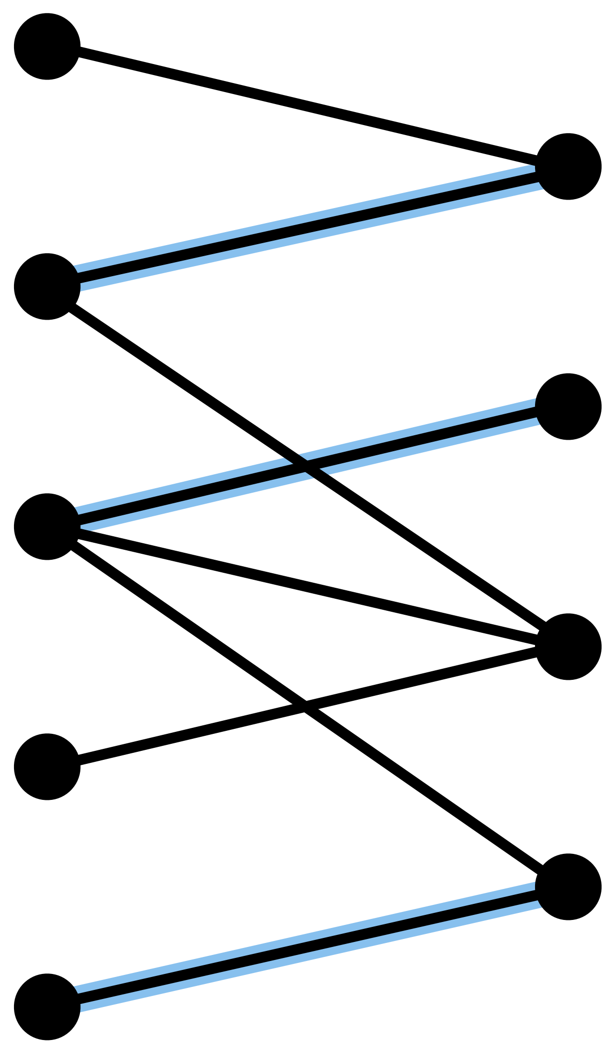

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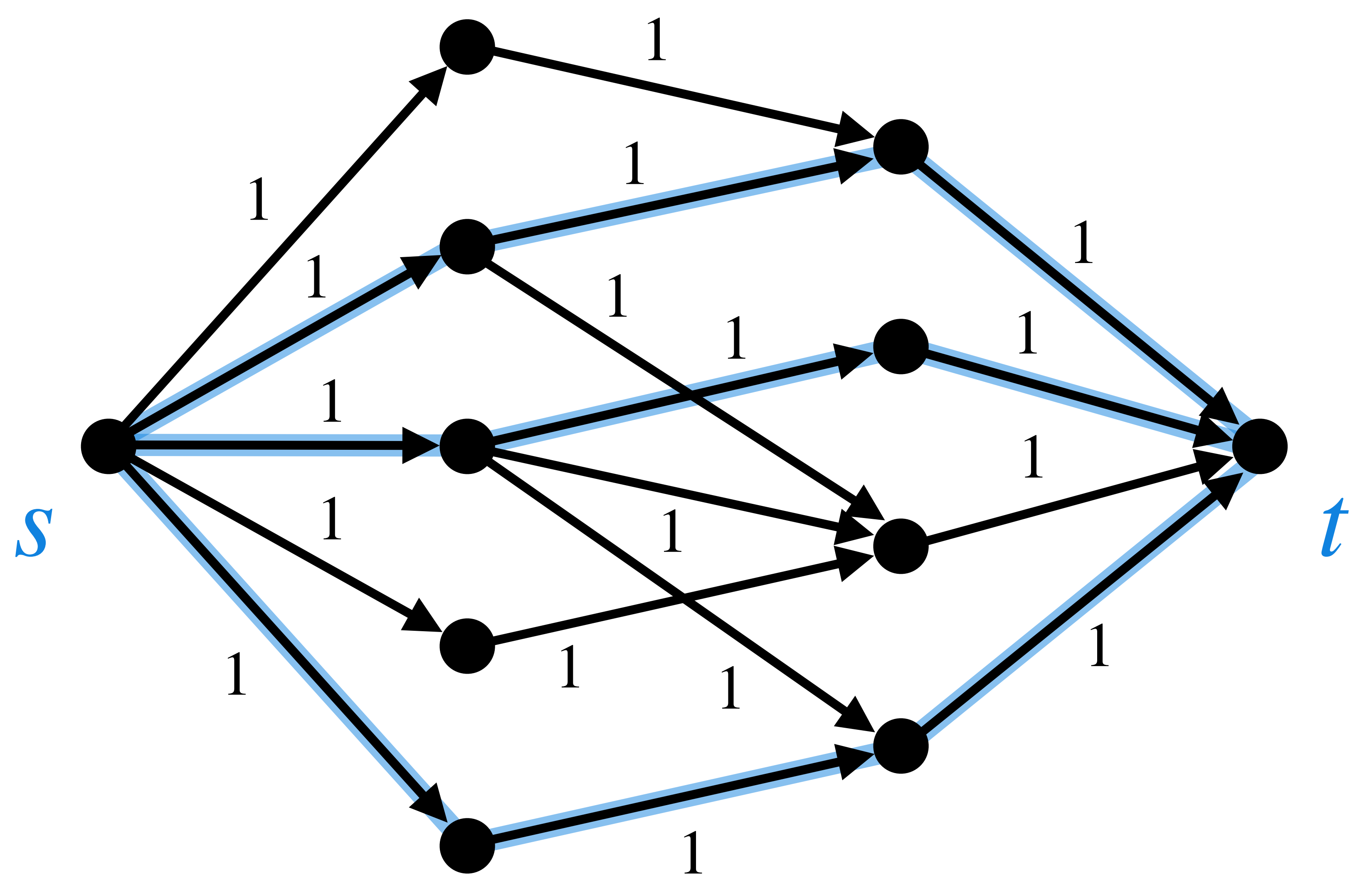

Bipartite Matching to Flow: Summary

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Let $G = (V, E)$ be a bipartite graph with partition (L, R) and let $G' = (V', E')$ be its corresponding flow network.

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Bipartite Matching to Flow: Summary

Let $G = (V, E)$ be a bipartite graph with partition (L, R) and let $G' = (V', E')$ be its corresponding flow network.

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Claim: If f is an **integer-valued** flow in G', then $\exists$ a matching M of size $|f|$ in G.

Max Bipartite Matching to Max Flow

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Max Bipartite Matching to Max Flow

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Max Bipartite Matching to Max Flow

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What if the flow produced has fractional values?